

Transformers



Some history

Historically, the first electrical power distribution system developed by Edison in 1880s was transmitting DC. It was designed for low voltages (safety and difficulties in voltage conversion); therefore, high currents were needed to be generated and transmitted to deliver necessary power. This system suffered significant energy losses!

The second generation of power distribution systems (what we are still using) was proposed by Tesla few years later. His idea was to generate AC power of any convenient voltage, step up the voltage for transmission (higher voltage implies lower current and, thus, lower losses), transmit AC power with small losses, and finally step down its voltage for consumption. Since power loss is proportional to the square of the current transmitted, raising the voltage, say, by the factor of 10 would decrease the current by the same factor (to deliver the same amount of energy) and, therefore, reduce losses by factor of 100.

The step up and step down voltage conversion was based on the use of transformers.



Preliminary considerations

A transformer is a device that converts one AC voltage to another AC voltage at the same frequency. It consists of one or more coil(s) of wire wrapped around a common ferromagnetic core. These coils are usually not connected electrically together. However, they are connected through the common magnetic flux confined to the core.

Assuming that the transformer has at least two windings, one of them (primary) is connected to a source of AC power; the other (secondary) is connected to the loads.



The invention of a transformer can be attributed to Faraday, who in 1831 used its principle to demonstrate electromagnetic induction foreseen no practical applications of his demonstration. ☹

Russian engineer Yablochkov in 1876 invented a lighting system based on a set of induction coils, which acted as a transformer.



More history

Gaulard and Gibbs first exhibited a device with an open iron core called a 'secondary generator' in London in 1882 and then sold the idea to a company Westinghouse. They also exhibited their invention in Turin in 1884, where it was adopted for an electric lighting system.

In 1885, William Stanley, an engineer for Westinghouse, built the first commercial transformer after George Westinghouse had bought Gaulard and Gibbs' patents. The core was made from interlocking E-shaped iron plates. This design was first used commercially in 1886.

Hungarian engineers Zipernowsky, Bláthy and Déri created the efficient "ZBD" closed-core model in 1885 based on the design by Gaulard and Gibbs. Their patent application made the first use of the word "transformer".

Another Russian engineer Dolivo-Dobrovolsky developed the first three-phase transformer in 1889.

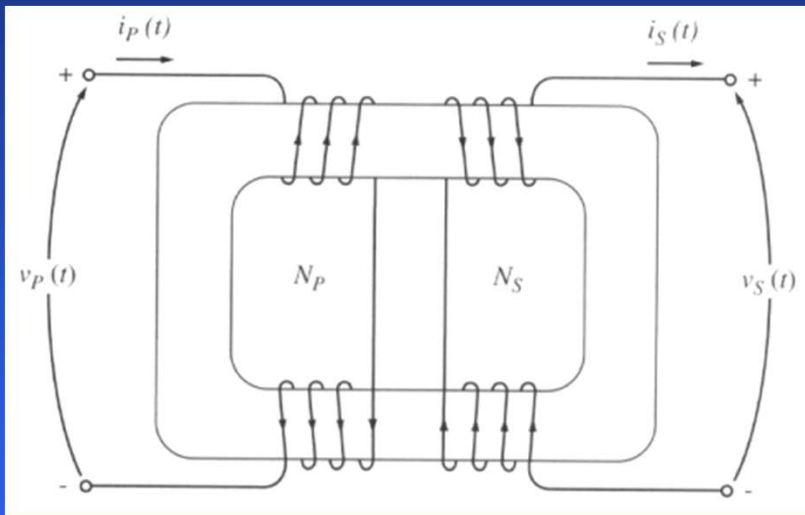
Finally, in 1891 Nikola Tesla invented the Tesla coil, an air-cored, dual-tuned resonant transformer for generating very high voltages at high frequency.



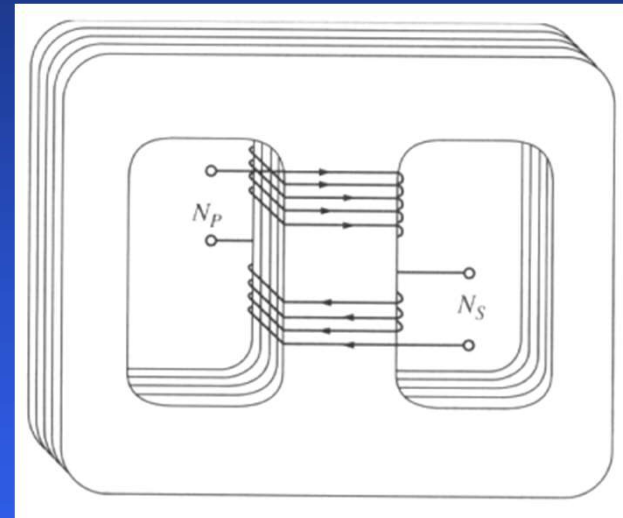
Types and construction

Power transformers

Core form ← → Shell form



Windings are wrapped around two sides of a laminated square core.



Windings are wrapped around the center leg of a laminated core.

Usually, windings are wrapped on top of each other to decrease flux leakage and, therefore, increase efficiency.

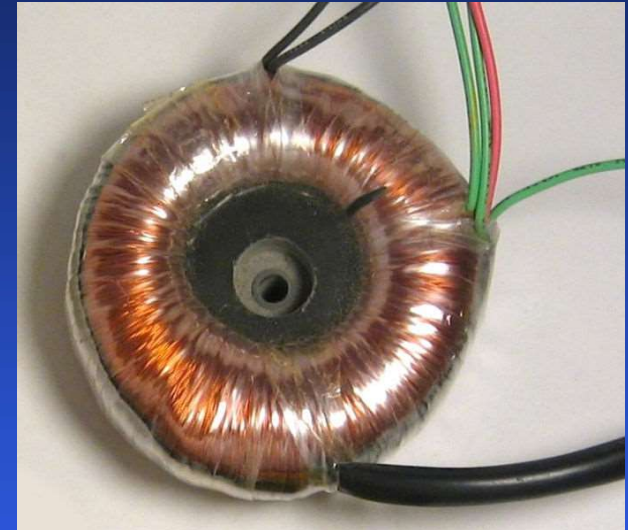


Types and construction



Laminated steel cores

Lamination
types



Toroidal steel cores

Efficiency of transformers with toroidal cores is usually higher.



Types and construction

Power transformers used in power distribution systems are sometimes referred as follows:

A power transformer connected to the output of a generator and used to step its voltage up to the transmission level (110 kV and higher) is called a unit transformer.

A transformer used at a substation to step the voltage from the transmission level down to the distribution level (2.3 ... 34.5 kV) is called a substation transformer.

A transformer converting the distribution voltage down to the final level (110 V, 220 V, etc.) is called a distribution transformer.

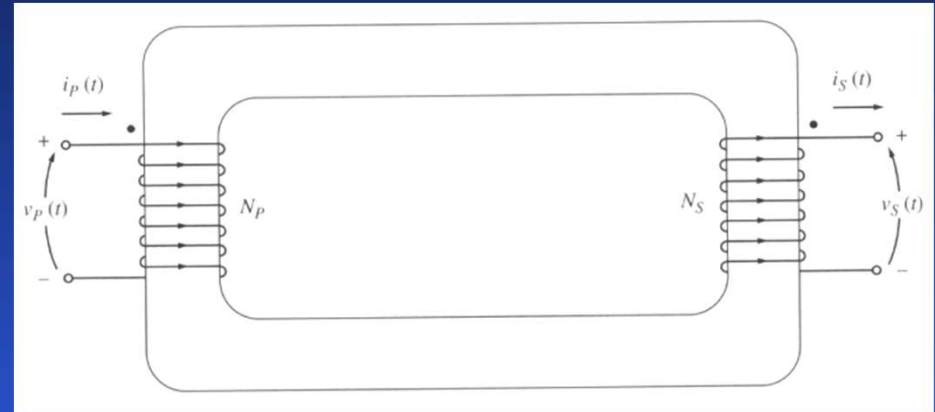
In addition to power transformers, other types of transformers are used.



Ideal transformer

We consider a lossless transformer with an input (primary) winding having N_p turns and a secondary winding of N_s turns.

The relationship between the voltage applied to the primary winding $v_p(t)$ and the voltage produced on the secondary winding $v_s(t)$ is



$$\frac{v_p(t)}{v_s(t)} = \frac{N_p}{N_s} = a \quad (4.8.1)$$

Here a is the turn ratio of the transformer.



Ideal transformer

The relationship between the primary $i_p(t)$ and secondary $i_s(t)$ currents is

$$\frac{i_p(t)}{i_s(t)} = \frac{1}{a} \quad (4.9.1)$$

In the phasor notation:

$$\frac{\mathbf{V}_p}{\mathbf{V}_s} = a \quad (4.9.2)$$

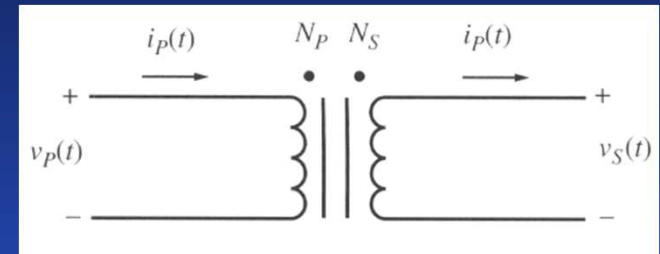
$$\frac{\mathbf{I}_p}{\mathbf{I}_s} = \frac{1}{a} \quad (4.9.3)$$

The phase angles of primary and secondary voltages are the same. The phase angles of primary and secondary currents are the same also. The ideal transformer changes magnitudes of voltages and currents but not their angles.



Ideal transformer

One winding's terminal is usually marked by a dot used to determine the polarity of voltages and currents.



If the voltage is positive at the dotted end of the primary winding at some moment of time, the voltage at the dotted end of the secondary winding will also be positive at the same time instance.

If the primary current flows into the dotted end of the primary winding, the secondary current will flow out of the dotted end of the secondary winding.



Power in an ideal transformer

Assuming that θ_p and θ_s are the angles between voltages and currents on the primary and secondary windings respectively, the power supplied to the transformer by the primary circuit is:

$$P_{in} = V_p I_p \cos \theta_p \quad (4.11.1)$$

The power supplied to the output circuits is

$$P_{out} = V_s I_s \cos \theta_s \quad (4.11.2)$$

Since ideal transformers do not affect angles between voltages and currents:

$$\theta_p = \theta_s = \theta \quad (4.11.3)$$

Both windings of an ideal transformer have the same power factor.



Power in an ideal transformer

Since for an ideal transformer the following holds:

$$V_s = \frac{V_p}{a}; \quad I_s = aI_p \quad (4.12.1)$$

Therefore:

$$P_{out} = V_s I_s \cos \theta = \frac{V_p}{a} a I_p \cos \theta = V_p I_p \cos \theta = P_{in} \quad (4.12.2)$$

The output power of an ideal transformer equals to its input power – to be expected since assumed no loss. Similarly, for reactive and apparent powers:

$$Q_{out} = V_s I_s \sin \theta = V_p I_p \sin \theta = Q_{in} \quad (4.12.3)$$

$$S_{out} = V_s I_s = V_p I_p = S_{in} \quad (4.12.4)$$



Impedance transformation

The impedance is defined as a following ratio of phasors:

$$Z_L = \mathbf{V}_L / \mathbf{I}_L \quad (4.13.1)$$

A transformer changes voltages and currents and, therefore, an apparent impedance of the load that is given by

$$Z_L = \mathbf{V}_s / \mathbf{I}_s \quad (4.13.2)$$

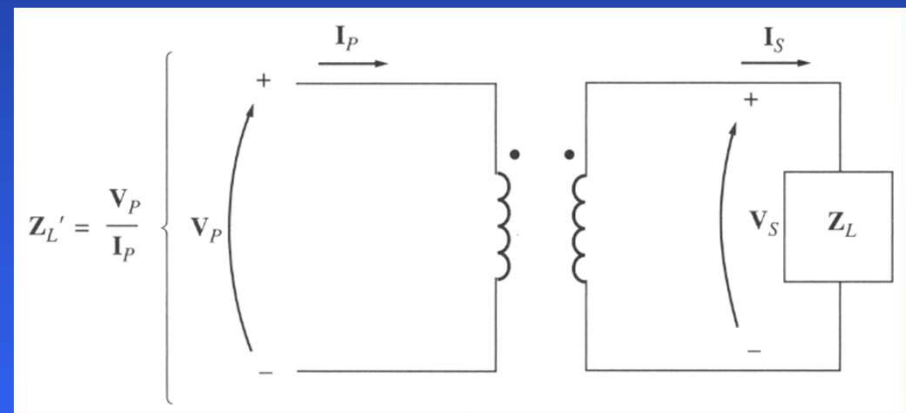
The apparent impedance of the primary circuit is:

$$Z_L' = \mathbf{V}_p / \mathbf{I}_p \quad (4.13.3)$$

which is

$$Z_L' = \frac{\mathbf{V}_p}{\mathbf{I}_p} = \frac{a\mathbf{V}_s}{\mathbf{I}_s/a} = a^2 \frac{\mathbf{V}_s}{\mathbf{I}_s} = a^2 Z_L \quad (4.13.4)$$

It is possible to match magnitudes of impedances (load and a transmission line) by selecting a transformer with the proper turn ratio.



Analysis of circuits containing ideal transformers

A simple method to analyze a circuit containing an ideal transformer is by replacing the portion of the circuit on one side of the transformer by an equivalent circuit with the same terminal characteristics.

Next, we exclude the transformer from the circuit and solve it for voltages and currents.

The solutions obtained for the portion of the circuit that was not replaced will be the correct values of voltages and currents of the original circuit.

Finally, the voltages and currents on the other side of the transformer (in the original circuit) can be found by considering the transformer's turn ratio.

This process is called referring of transformer's sides.

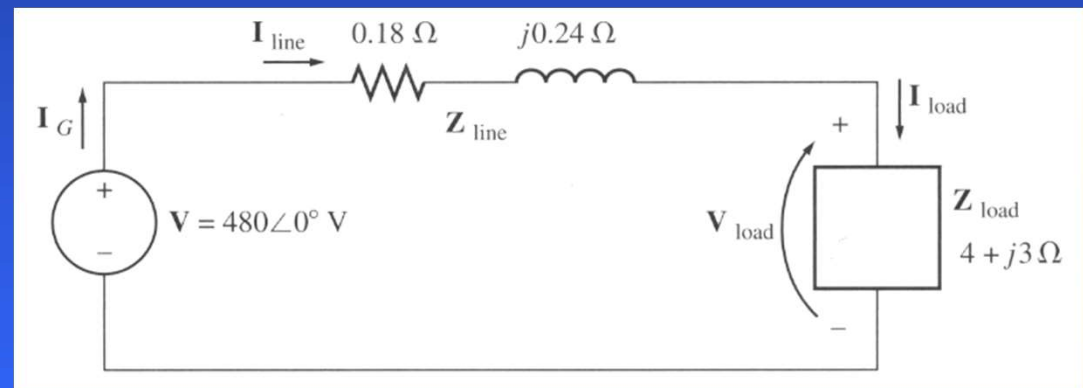


Analysis of circuits containing ideal transformers: Example

Example 4.1: A single-phase power system consists of a 480-V 60-Hz generator that is connected to the load $Z_{load} = 4 + j3 \Omega$ through the transmission line with $Z_{line} = 0.18 + j0.24 \Omega$. a) What is the voltage at the load? What are the transmission line losses? b) If a 1:10 step up transformer and a 10:1 step down transformer are placed at the generator and the load ends of the transmission line respectively, what are the new load voltage and the new transmission line losses?

a) Here:

$$\begin{aligned} \mathbf{I}_G = \mathbf{I}_{line} = \mathbf{I}_{load} &= \frac{\mathbf{V}}{\mathbf{Z}_{line} + \mathbf{Z}_{load}} \\ &= \frac{480 \angle 0^\circ}{0.18 + j0.24 + 4 + j3} \\ &= \frac{480 \angle 0^\circ}{5.29 \angle 37.8^\circ} = 90.8 \angle -37.8^\circ \text{ A} \end{aligned}$$



Analysis of circuits containing ideal transformers: Example

Therefore, the load voltage:

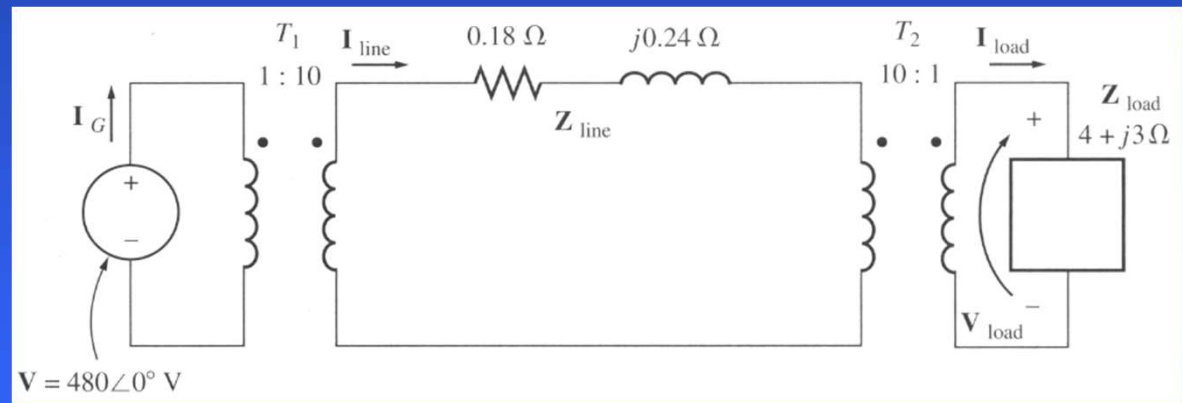
$$\mathbf{V}_{load} = \mathbf{I}_{load} \mathbf{Z}_{load} = (90.8 \angle -37.8^\circ)(4 + j3) = (90.8 \angle -37.8^\circ)(5 \angle 36.9^\circ) = 454 \angle -0.9^\circ \text{ V}$$

The line losses are:

$$P_{loss} = I_{line}^2 R_{line} = 90.8^2 \cdot 0.18 = 1484 \text{ W}$$

b) We will

- 1) eliminate transformer T_2 by referring the load over to the transmission line's voltage level.
- 2) Eliminate transformer T_1 by referring the transmission line's elements and the equivalent load at the transmission line's voltage over to the source side.



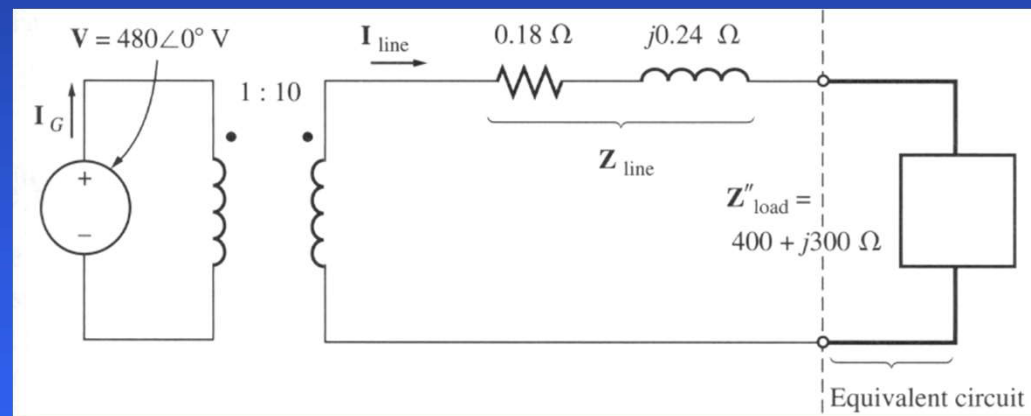
Analysis of circuits containing ideal transformers: Example

The load impedance when referred to the transmission line (while the transformer T_2 is eliminated) is:

$$\mathbf{Z}'_{load} = a_2^2 \mathbf{Z}_{load} = \left(\frac{10}{1} \right)^2 (4 + j3) = 400 + j300$$

The total impedance on the transmission line level is

$$\begin{aligned} \mathbf{Z}_{eq} &= \mathbf{Z}_{line} + \mathbf{Z}'_{load} \\ &= 400.18 + j300.24 \\ &= 500.3 \angle 36.88^\circ \Omega \end{aligned}$$



The total impedance is now referred across T_1 to the source's voltage level:

$$\mathbf{Z}'_{eq} = a_1^2 \mathbf{Z}_{eq} = \left(\frac{1}{10} \right)^2 (500.3 \angle 36.88^\circ) = 5.003 \angle 36.88^\circ \Omega$$



Analysis of circuits containing ideal transformers: Example

The generator's current is

$$\begin{aligned} \mathbf{I}_G &= \frac{\mathbf{V}}{\mathbf{Z}'_{eq}} = \frac{480 \angle 0^\circ}{5.003 \angle 36.88^\circ} \\ &= 95.94 \angle -36.88^\circ \text{ A} \end{aligned}$$

Knowing transformers' turn ratios, we can determine line and load currents:

$$\mathbf{I}_{line} = a_1 \mathbf{I}_G = 0.1 \cdot (95.94 \angle -36.88^\circ) = 9.594 \angle -36.88^\circ \text{ A}$$

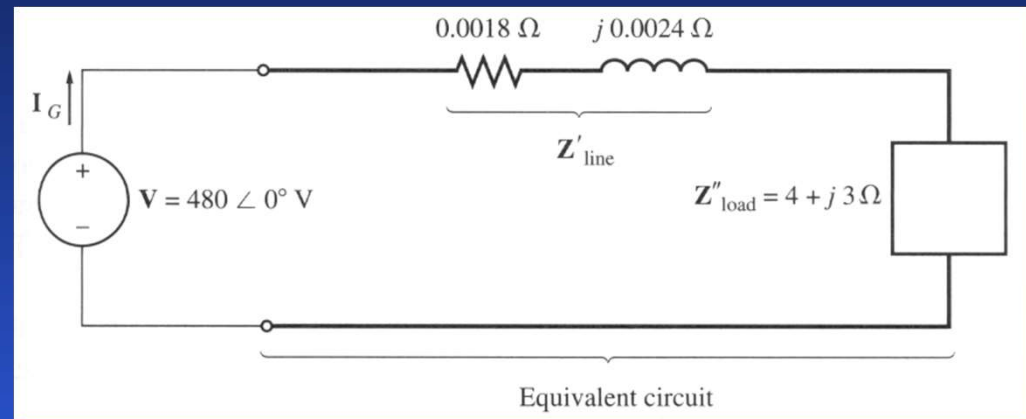
$$\mathbf{I}_{load} = a_2 \mathbf{I}_{line} = 10 \cdot (9.594 \angle -36.88^\circ) = 95.94 \angle -36.88^\circ \text{ A}$$

Therefore, the load voltage is:

$$\mathbf{V}_{load} = \mathbf{I}_{load} \mathbf{Z}_{load} = (95.94 \angle -36.88^\circ)(5 \angle -36.87^\circ) = 479.7 \angle -0.01^\circ \text{ V}$$

The losses in the line are: $P_{loss} = I_{line}^2 R_{line} = 9.594^2 \cdot 0.18 = 16.7 \text{ W}$

Note: transmission line losses are reduced by a factor nearly 90, the load voltage is much closer to the generator's voltage – effects of increasing the line's voltage.



Theory of operation of real single-phase transformers

Real transformers approximate ideal ones to some degree.

The basis transformer operation can be derived from Faraday's law:

$$e_{ind} = \frac{d\lambda}{dt} \quad (4.19.1)$$

Here λ is the flux linkage in the coil across which the voltage is induced:

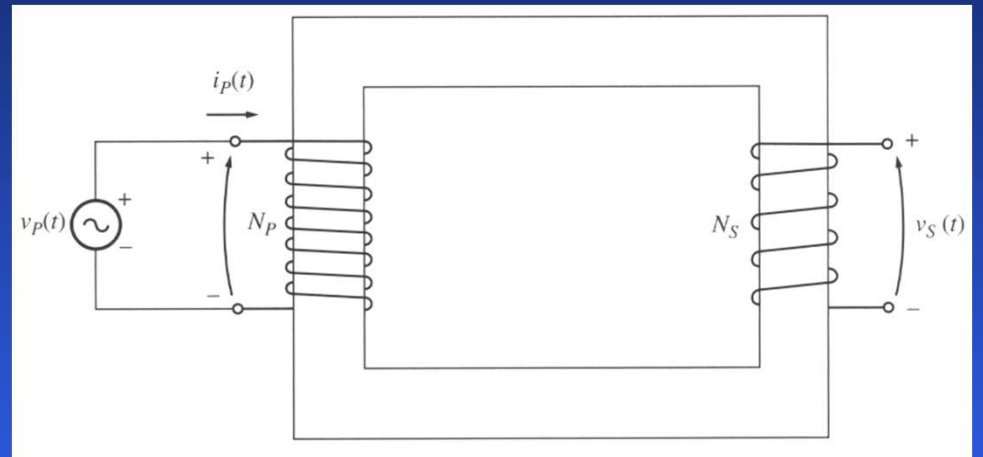
$$\lambda = \sum_{i=1}^N \phi_i$$

where ϕ_i is the flux passing through the i^{th} turn in a coil – slightly different for different turns. However, we may use an average flux per turn in the coil having N turns:

$$\bar{\phi} = \lambda/N \quad (4.19.3)$$

Therefore:

$$e_{ind} = N \frac{d\bar{\phi}}{dt} \quad (4.19.4)$$



(4.19.2)



The voltage ratio across a real transformer

If the source voltage $v_p(t)$ is applied to the primary winding, the average flux in the primary winding will be:

$$\bar{\phi} = \frac{1}{N_p} \int v_p(t) dt \quad (4.20.1)$$

A portion of the flux produced in the primary coil passes through the secondary coil (mutual flux); the rest is lost (leakage flux):

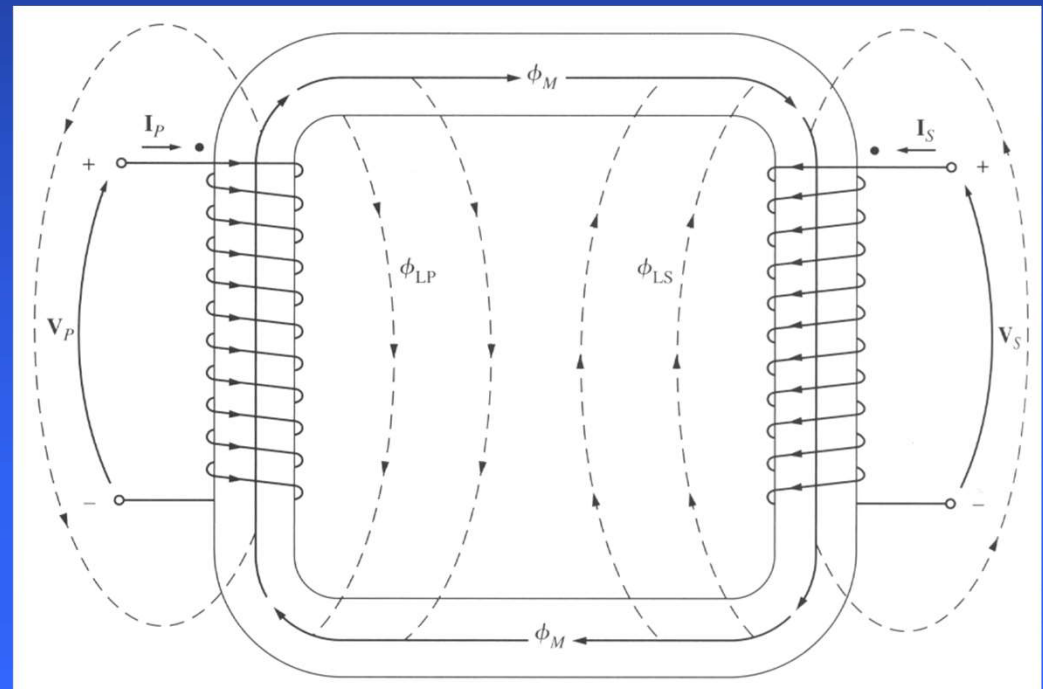
$$\bar{\phi}_p = \phi_m + \phi_{Lp} \quad (4.20.2)$$

average primary flux mutual flux

Similarly, for the secondary coil:

$$\bar{\phi}_s = \phi_m + \phi_{Ls} \quad (4.20.3)$$

Average secondary flux



The voltage ratio across a real transformer

From the Faraday's law, the primary coil's voltage is:

$$v_p(t) = N_p \frac{d\bar{\phi}_p}{dt} = N_p \frac{d\phi_m}{dt} + N_p \frac{d\phi_{Lp}}{dt} = e_p(t) + e_{Lp}(t) \quad (4.21.1)$$

The secondary coil's voltage is:

$$v_s(t) = N_s \frac{d\bar{\phi}_s}{dt} = N_s \frac{d\phi_m}{dt} + N_s \frac{d\phi_{Ls}}{dt} = e_s(t) + e_{Ls}(t) \quad (4.21.2)$$

The primary and secondary voltages due to the mutual flux are:

$$e_p(t) = N_p \frac{d\phi_m}{dt} \quad (4.21.3)$$

$$e_s(t) = N_s \frac{d\phi_m}{dt} \quad (4.21.4)$$

Combining the last two equations:

$$\frac{e_p(t)}{N_p} = \frac{d\phi_m}{dt} = \frac{e_s(t)}{N_s} \quad (4.21.5)$$



The voltage ratio across a real transformer

Therefore:

$$\frac{e_p(t)}{e_s(t)} = \frac{N_p}{N_s} = a \quad (4.22.1)$$

That is, the ratio of the primary voltage to the secondary voltage both caused by the mutual flux is equal to the turns ratio of the transformer.

For well-designed transformers:

$$\phi_m \approx \phi_{Lp}; \quad \phi_m \approx \phi_{Ls} \quad (4.22.2)$$

Therefore, the following approximation normally holds:

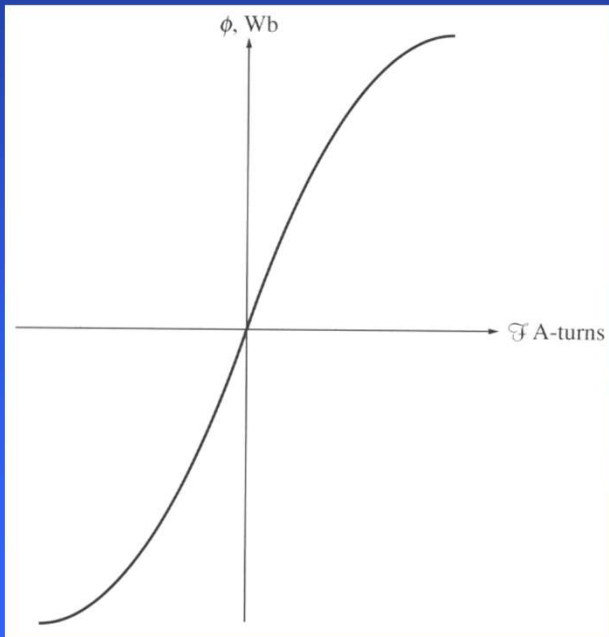
$$\frac{v_p(t)}{v_s(t)} \approx \frac{N_p}{N_s} \approx a \quad (4.22.3)$$



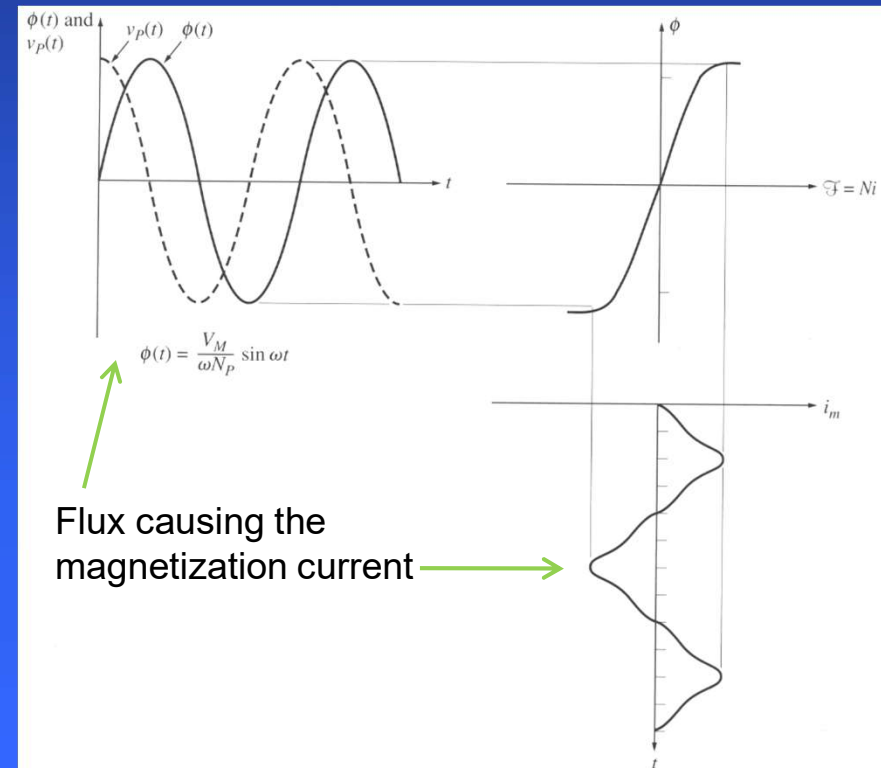
The magnetization current in a real transformer

Even when no load is connected to the secondary coil of the transformer, a current will flow in the primary coil. This current consists of:

1. The magnetization current i_m needed to produce the flux in the core;
2. The core-loss current i_{h+e} hysteresis and eddy current losses.



Typical magnetization curve



The magnetization current in a real transformer

Ignoring flux leakage and assuming time-harmonic primary voltage, the average flux is:

$$\bar{\phi} = \frac{1}{N_p} \int v_p(t) dt = \frac{1}{N_p} \int V_m \cos \omega t dt = \frac{V_m}{\omega N_p} \sin \omega t \quad [Wb] \quad (4.24.1)$$

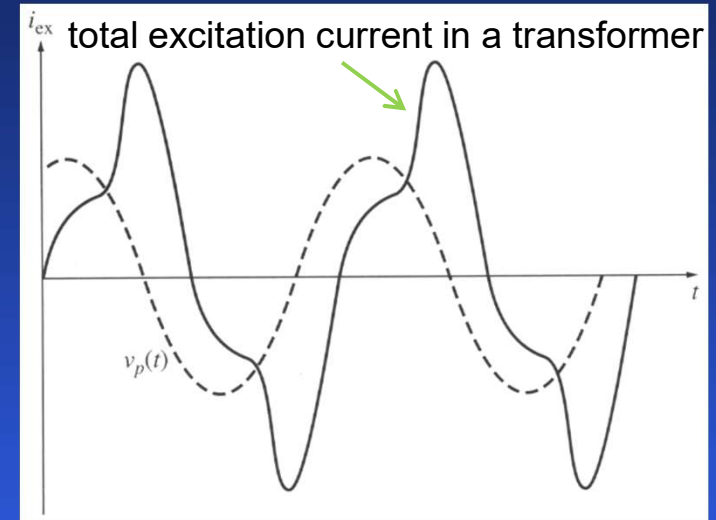
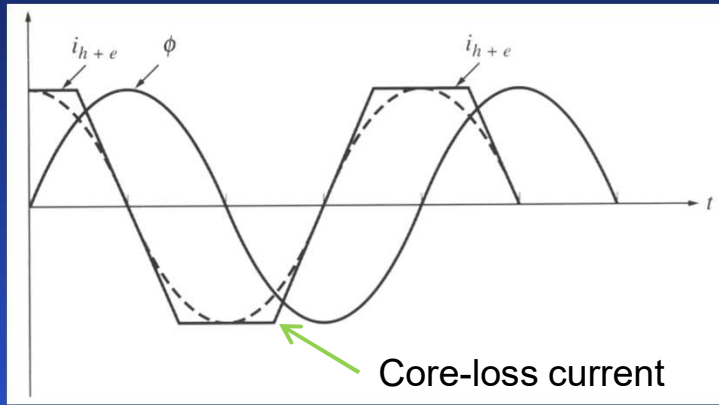
If the values of current are comparable to the flux they produce in the core, it is possible to sketch a magnetization current. We observe:

1. Magnetization current is not sinusoidal: there are high frequency components;
2. Once saturation is reached, a small increase in flux requires a large increase in magnetization current;
3. Magnetization current (its fundamental component) lags the voltage by 90°;
4. High-frequency components of the current may be large in saturation.

Assuming a sinusoidal flux in the core, the eddy currents will be largest when flux passes zero.



The magnetization current in a real transformer



Core-loss current is:

1. Nonlinear due to nonlinear effects of hysteresis;
2. In phase with the voltage.

The total no-load current in the core is called the excitation current of the transformer:

$$i_{ex} = i_m + i_{h+e} \quad (4.25.1)$$



The current ratio on a transformer

If a load is connected to the secondary coil, there will be a current flowing through it.

A current flowing into the dotted end of a winding produces a positive magnetomotive force F :

$$F_p = N_p i_p \quad (4.26.1)$$

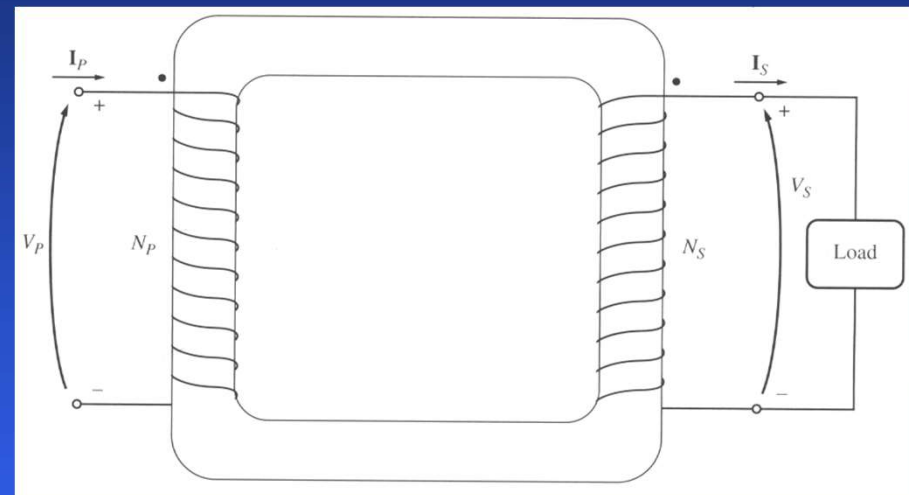
$$F_s = N_s i_s \quad (4.26.2)$$

The net magnetomotive force in the core

$$F_{net} = N_p i_p - N_s i_s = \phi \mathfrak{R} \quad (4.26.3)$$

where \mathfrak{R} is the reluctance of the transformer core. For well-designed transformer cores, the reluctance is very small if the core is not saturated. Therefore:

$$F_{net} = N_p i_p - N_s i_s \approx 0 \quad (4.26.4)$$



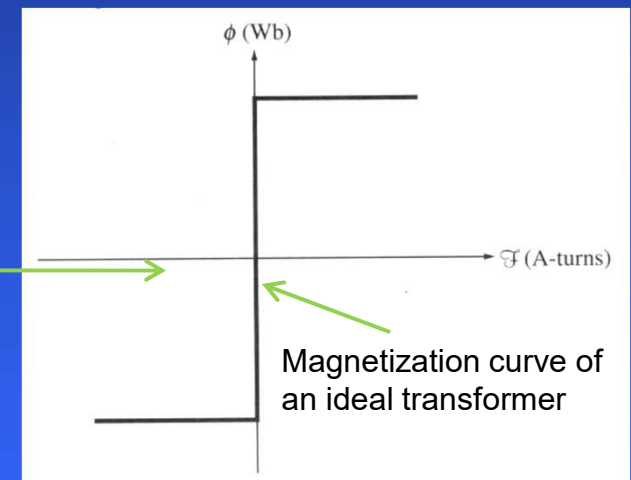
The current ratio on a transformer

The last approximation is valid for well-designed unsaturated cores. Therefore:

$$N_p i_p \approx N_s i_s \Rightarrow \frac{i_p}{i_s} \approx \frac{N_s}{N_p} = \frac{1}{a} \quad (4.27.1)$$

An ideal transformer (unlike the real one) can be characterized as follows:

1. The core has no hysteresis or eddy currents.
2. The magnetization curve is
3. The leakage flux in the core is zero.
4. The resistance of the windings is zero.



The transformer's equivalent circuit

To model a real transformer accurately, we need to account for the following losses:

1. Copper losses – resistive heating in the windings: I^2R .
2. Eddy current losses – resistive heating in the core: proportional to the square of voltage applied to the transformer.
3. Hysteresis losses – energy needed to rearrange magnetic domains in the core: nonlinear function of the voltage applied to the transformer.
4. Leakage flux – flux that escapes from the core and flux that passes through one winding only.

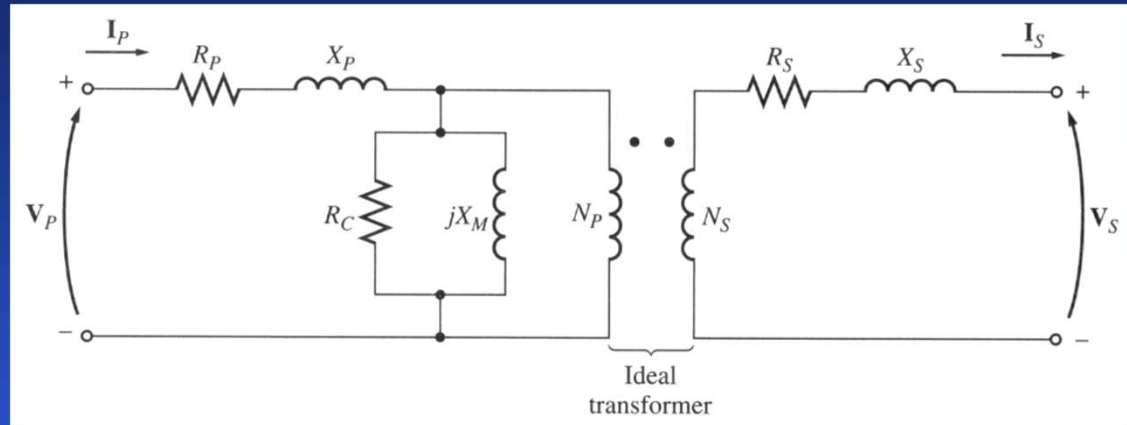


The exact equivalent circuit of a real transformer

Copper losses are modeled by the resistors R_p and R_s .

Leakage flux in a primary winding produces the voltage:

$$e_{Lp}(t) = N_p \frac{d\phi_{Lp}}{dt} \quad (4.29.1)$$



Since much of the leakage flux pass through air, and air has a constant reluctance that is much higher than the core reluctance, the primary coil's leakage flux is:

$$\phi_{Lp} = \underset{\substack{\uparrow \\ \text{permeance of flux path}}}{\mathcal{P}} N_p i_p \quad (4.29.2)$$

Therefore:

$$e_{Lp}(t) = N_p \frac{d}{dt} (\mathcal{P} N_p i_p) = N_p^2 \mathcal{P} \frac{di_p}{dt} \quad (4.29.3)$$



The exact equivalent circuit of a real transformer

Recognizing that the self-inductance of the primary coil is

$$L_p = N_p^2 \mathcal{P} \quad (4.30.1)$$

The induced voltages are:

Primary coil:

$$e_{L_p}(t) = L_p \frac{di_p}{dt} \quad (4.30.2)$$

Secondary coil:

$$e_{L_s}(t) = L_s \frac{di_s}{dt} \quad (4.30.3)$$

The leakage flux can be modeled by primary and secondary inductors.

The magnetization current can be modeled by a reactance X_M connected across the primary voltage source.

The core-loss current can be modeled by a resistance R_C connected across the primary voltage source.

Both currents are nonlinear; therefore, X_M and R_C are just approximations.



The exact equivalent circuit of a real transformer

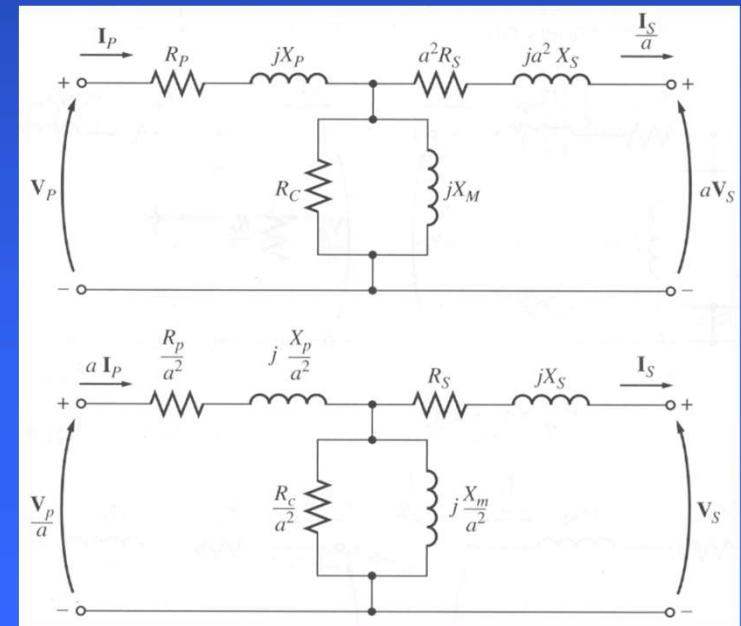
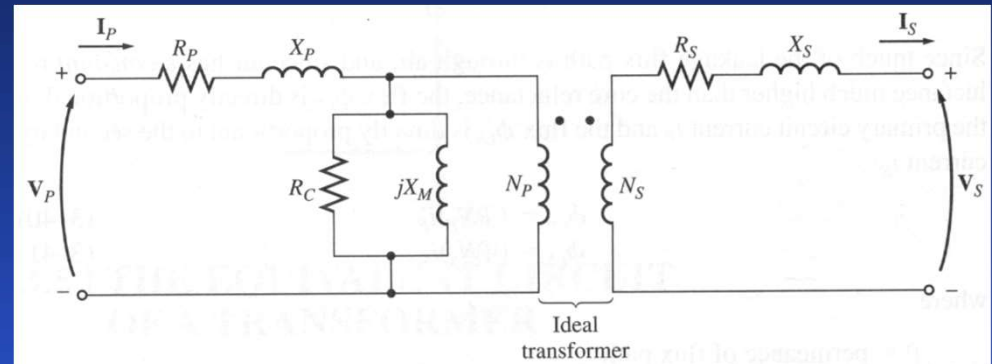
The transformer's equivalent circuit

However, the exact circuit is not very practical.

Therefore, the equivalent circuit is usually referred to the primary side or the secondary side of the transformer.

Equivalent circuit of the transformer referred to its primary side.

Equivalent circuit of the transformer referred to its secondary side.



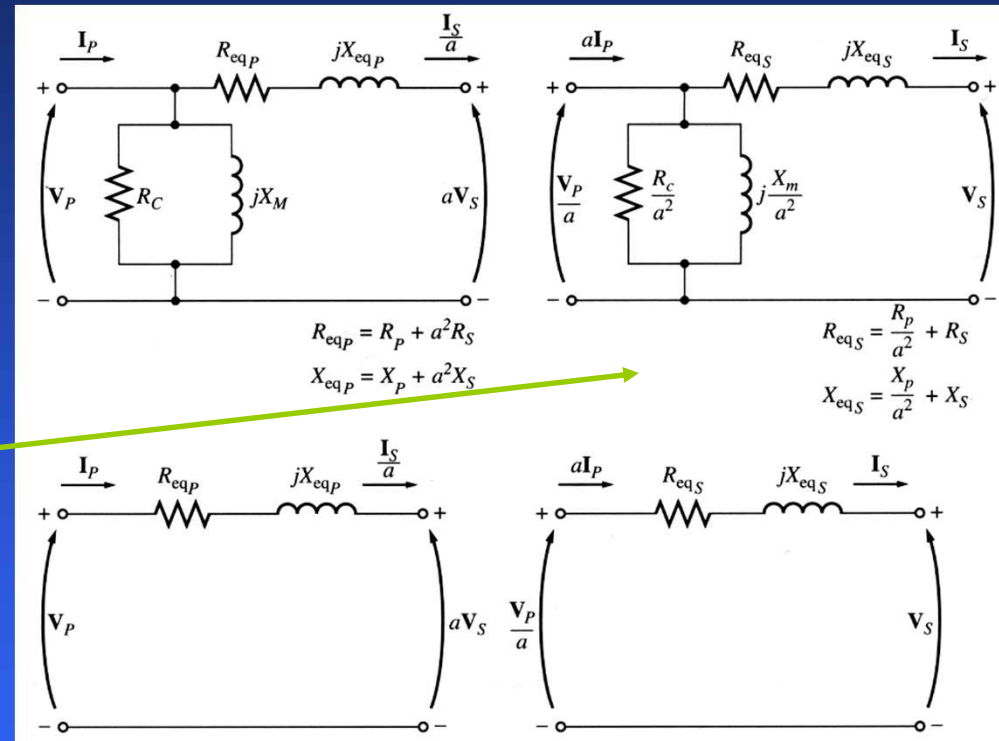
Approximate equivalent circuit of a transformer

For many practical applications, approximate models of transformers are used.

Referred to the primary side.

Referred to the secondary side.

Without an excitation branch referred to the primary side.



The values of components of the transformer model can be determined experimentally by an open-circuit test or by a short-circuit test.

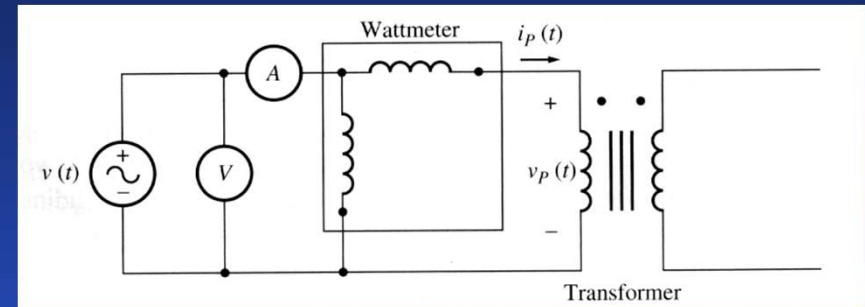
Without an excitation branch referred to the secondary side.



Determining the values of components

The open-circuit test.

Full line voltage is applied to the primary side of the transformer. The input voltage, current, and power are measured.



From this information, the power factor of the input current and the magnitude and the angle of the excitation impedance can be determined.

To evaluate R_C and X_M , we determine the conductance of the core-loss resistor is:

$$G_C = \frac{1}{R_C} \quad (4.33.1)$$

The susceptance of the magnetizing inductor is:

$$B_M = \frac{1}{X_M} \quad (4.33.2)$$



Determining the values of components

Since both elements are in parallel, their admittances add. Therefore, the total excitation admittance is:

$$Y_E = G_C - jB_M = \frac{1}{R_C} - j\frac{1}{X_M} \quad (4.34.1)$$

The magnitude of the excitation admittance in the open-circuit test is:

$$|Y_E| = \frac{I_{oc}}{V_{oc}} \quad (4.34.2)$$

The angle of the admittance in the open-circuit test can be found from the circuit power factor (PF):

$$\cos \theta = PF = \frac{P_{oc}}{V_{oc} I_{oc}} \quad (4.34.3)$$



Determining the values of components

In real transformers, the power factor is always lagging, so the angle of the current always lags the angle of the voltage by θ degrees. The admittance is:

$$Y_E = \frac{I_{oc}}{V_{oc}} \angle -\theta = \frac{I_{oc}}{V_{oc}} \angle -\cos^{-1} PF \quad (4.35.1)$$

Therefore, it is possible to determine values of R_C and X_M in the open-circuit test.



Determining the values of components

The short-circuit test.

Fairly low input voltage is applied to the primary side of the transformer. This voltage is adjusted until the current in the secondary winding equals to its rated value.

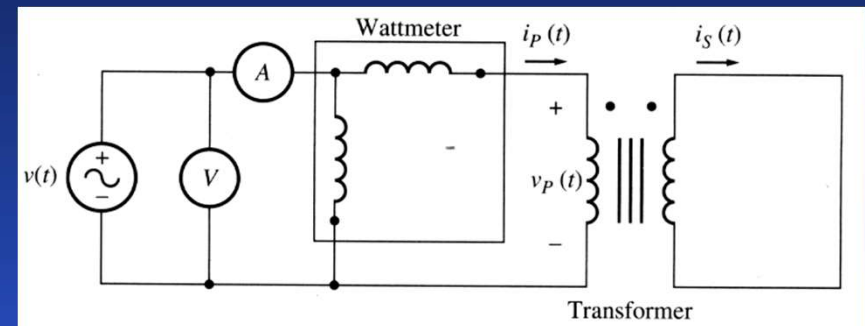
The input voltage, current, and power are again measured.

Since the input voltage is low, the current flowing through the excitation branch is negligible; therefore, all the voltage drop in the transformer is due to the series elements in the circuit. The magnitude of the series impedance referred to the primary side of the transformer is:

$$|Z_{SE}| = \frac{V_{SC}}{I_{SC}} \quad (4.36.1)$$

The power factor of the current is given by:

$$PF = \cos \theta = \frac{P_{SC}}{V_{SC} I_{SC}} \quad (4.36.2)$$



Determining the values of components

Therefore:

$$Z_{SE} = \frac{V_{SC} \angle 0^\circ}{I_{SC} \angle -\theta^\circ} = \frac{V_{SC}}{I_{SC}} \angle \theta^\circ \quad (4.37.1)$$

Since the serial impedance Z_{SE} is equal to

$$Z_{SE} = R_{eq} + jX_{eq} \quad (4.37.2)$$

$$Z_{SE} = (R_p + a^2 R_s) + j(X_p + a^2 X_s) \quad (4.37.3)$$

it is possible to determine the total series impedance referred to the primary side of the transformer. However, there is no easy way to split the series impedance into primary and secondary components.

The same tests can be performed on the secondary side of the transformer. The results will yield the equivalent circuit impedances referred to the secondary side of the transformer.



Determining the values of components: Example

Example 4.2: We need to determine the equivalent circuit impedances of a 20 kVA, 8000/240 V, 60 Hz transformer. The open-circuit and short-circuit tests led to the following data:

$V_{OC} = 8000 \text{ V}$	$V_{SC} = 489 \text{ V}$
$I_{OC} = 0.214 \text{ A}$	$I_{SC} = 2.5 \text{ A}$
$P_{OC} = 400 \text{ W}$	$P_{SC} = 240 \text{ W}$

The power factor during the open-circuit test is

$$PF = \cos \theta = \frac{P_{OC}}{V_{OC} I_{OC}} = \frac{400}{8000 \cdot 0.214} = 0.234 \text{ lagging}$$

The excitation admittance is

$$Y_E = \frac{I_{OC}}{V_{OC}} \angle -\cos^{-1} PF = \frac{0.214}{8000} \angle -\cos^{-1} 0.234 = 0.0000063 - j0.0000261 = \frac{1}{R_C} - j\frac{1}{X_M}$$



Determining the values of components: Example

Therefore: $R_C = \frac{1}{0.0000063} = 159 \text{ k}\Omega$; $X_M = \frac{1}{0.0000261} = 38.3 \text{ k}\Omega$

The power factor during the short-circuit test is

$$PF = \cos \theta = \frac{P_{SC}}{V_{SC} I_{SC}} = \frac{240}{489 \cdot 2.5} = 0.196 \text{ lagging}$$

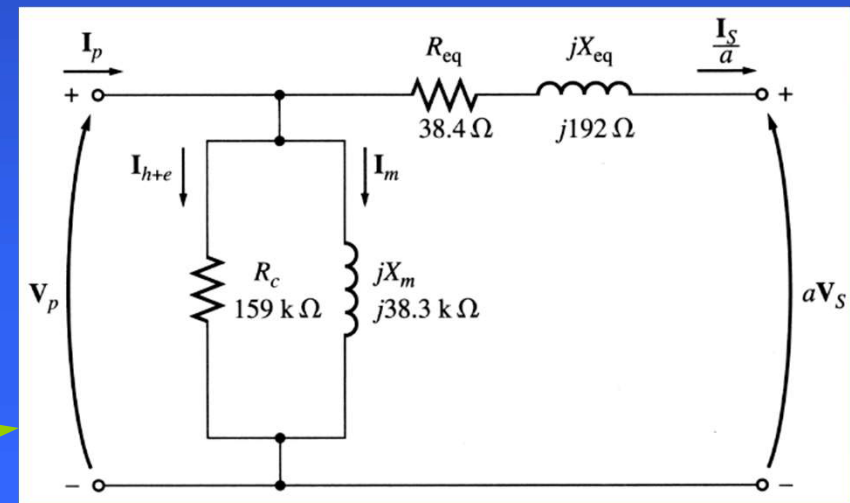
The series impedance is given by

$$\begin{aligned} Z_{SE} &= \frac{V_{SC}}{I_{SC}} \angle \cos^{-1} PF = \frac{489}{2.5} \angle 78.7^\circ \\ &= 38.4 + j192 \Omega \end{aligned}$$

Therefore:

$$R_{eq} = 38.3 \Omega; \quad X_{eq} = 192 \Omega$$

The equivalent circuit



The per-unit system

Another approach to solve circuits containing transformers is the per-unit system. Impedance and voltage-level conversions are avoided. Also, machine and transformer impedances fall within fairly narrow ranges for each type and construction of device while the per-unit system is employed.

The voltages, currents, powers, impedances, and other electrical quantities are measured as fractions of some base level instead of conventional units.

$$\text{Quantity per unit} = \frac{\text{actual value}}{\text{base value of quantity}} \quad (4.40.1)$$

Usually, two base quantities are selected to define a given per-unit system. Often, such quantities are voltage and power (or apparent power). In a 1-phase system:

$$P_{base}, Q_{base}, \text{ or } S_{base} = V_{base} I_{base} \quad (4.40.2)$$

$$Z_{base} = \frac{V_{base}}{I_{base}} = \frac{(V_{base})^2}{S_{base}} \quad (4.40.3)$$



The per-unit system

$$Y_{base} = \frac{I_{base}}{V_{base}} \quad (4.41.1)$$

Once the base values of P (or S) and V are selected, all other base values can be computed from the above equations.

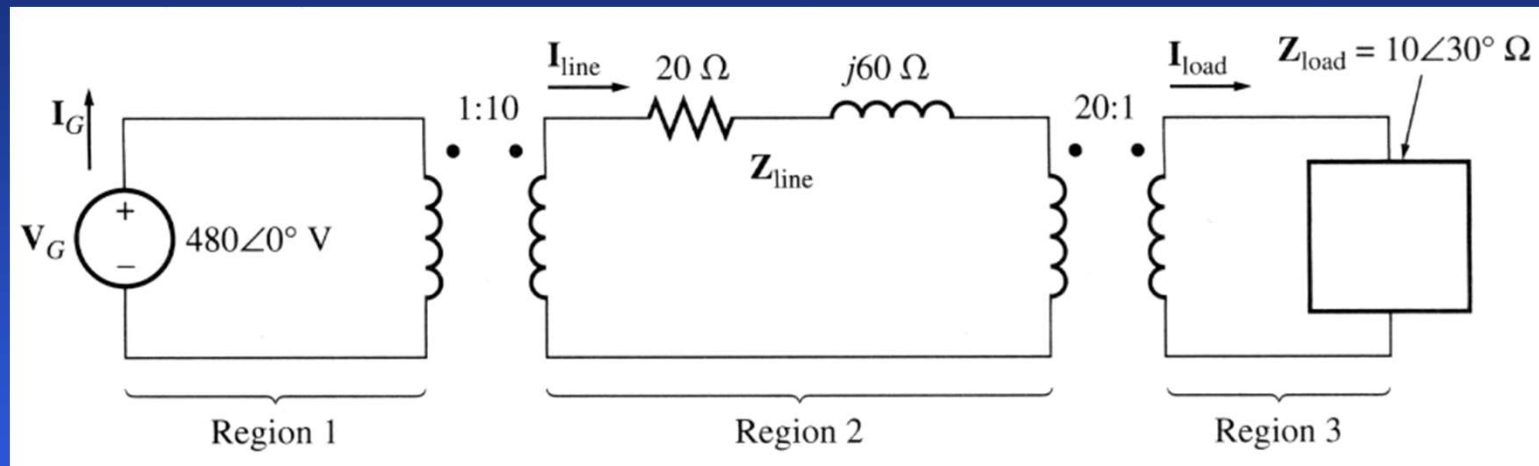
In a power system, a base apparent power and voltage are selected at the specific point in the system. Note that a transformer has no effect on the apparent power of the system, since the apparent power into a transformer equals the apparent power out of a transformer. As a result, the base apparent power remains constant everywhere in the power system.

On the other hand, voltage (and, therefore, a base voltage) changes when it goes through a transformer according to its turn ratio. Therefore, the process of referring quantities to a common voltage level is done automatically in the per-unit system.



The per-unit system: Example

Example 4.3: A simple power system is given by the circuit:



The generator is rated at 480 V and 10 kVA.

- Find the base voltage, current, impedance, and apparent power at every points in the power system;
- Convert the system to its per-unit equivalent circuit;
- Find the power supplied to the load in this system;
- Find the power lost in the transmission line (Region 2).



The per-unit system: Example

a. In the generator region: $V_{base\ 1} = 480\ V$ and $S_{base} = 10\ kVA$

$$I_{base\ 1} = \frac{S_{base\ 1}}{V_{base\ 1}} = \frac{10\ 000}{480} = 20.83\ A$$

$$Z_{base\ 1} = \frac{V_{base\ 1}}{I_{base\ 1}} = \frac{480}{20.83} = 23.04\ \Omega$$

The turns ratio of the transformer T_1 is $a_1 = 0.1$; therefore, the voltage in the transmission line region is

$$V_{base\ 2} = \frac{V_{base\ 1}}{a_1} = \frac{480}{0.1} = 4800\ V$$

The other base quantities are



The per-unit system: Example

$$S_{base 2} = 10 \text{ kVA}$$

$$I_{base 2} = \frac{10\,000}{4800} = 2.083 \text{ A}$$

$$Z_{base 2} = \frac{4800}{2.083} = 2304 \Omega$$

The turns ratio of the transformer T_2 is $a_2 = 20$; therefore, the voltage in the load region is

$$V_{base 3} = \frac{V_{base 2}}{a_2} = \frac{4800}{20} = 240 \text{ V}$$

The other base quantities are



The per-unit system: Example

$$S_{base3} = 10 \text{ kVA}$$

$$I_{base3} = \frac{10\,000}{240} = 41.67 \text{ A}$$

$$Z_{base3} = \frac{240}{41.67} = 5.76 \Omega$$

b. To convert a power system to a per-unit system, each component must be divided by its base value in its region. The generator's per-unit voltage is

$$V_{G,pu} = \frac{480 \angle 0^\circ}{480} = 1.0 \angle 0^\circ \text{ pu}$$

The transmission line's per-unit impedance is

$$Z_{line,pu} = \frac{20 + j60}{2304} = 0.0087 + j0.026 \text{ pu}$$

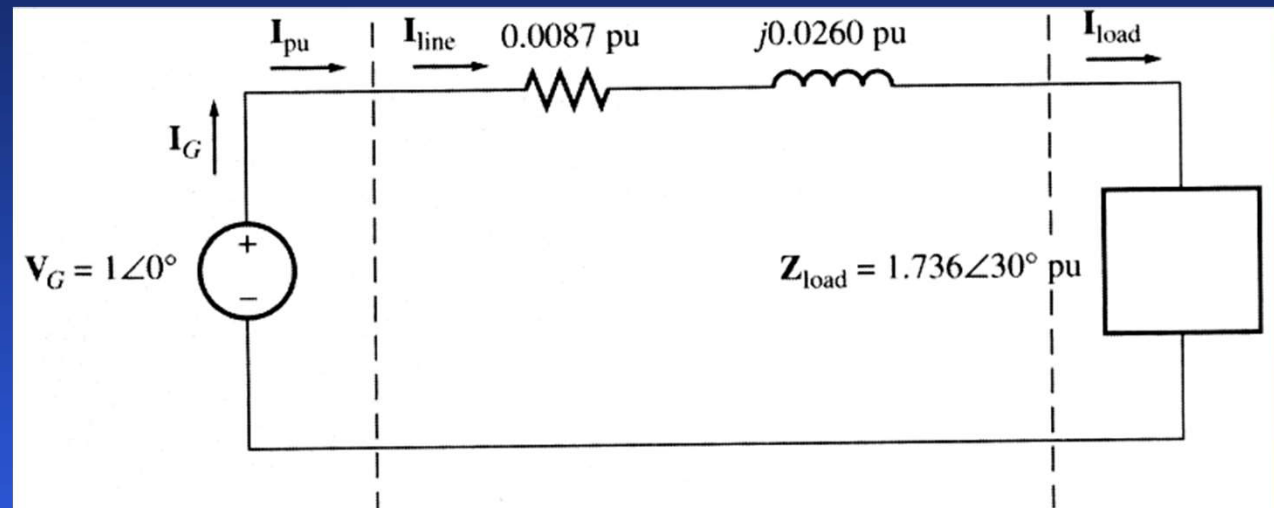


The per-unit system: Example

The load's per-unit impedance is

$$Z_{load, pu} = \frac{10 \angle 30^\circ}{5.76} = 1.736 \angle 30^\circ pu$$

The per-unit equivalent circuit



c. The current flowing in this per-unit power system is

$$I_{pu} = \frac{V_{pu}}{Z_{tot, pu}} = \frac{1 \angle 0^\circ}{0.0087 + j0.026 + 1.736 \angle 30^\circ} = 0.569 \angle -30.6^\circ pu$$



The per-unit system: Example

Therefore, the per-unit power on the load is

$$P_{load, pu} = I_{pu}^2 R_{pu} = 0.569^2 \cdot 1.503 = 0.487$$

The actual power on the load is

$$P_{load} = P_{load, pu} S_{base} = 0.487 \cdot 10\,000 = 487\, W$$

d. The per-unit power lost in the transmission line is

$$P_{line, pu} = I_{pu}^2 R_{line, pu} = 0.569^2 \cdot 0.0087 = 0.00282$$

The actual power lost in the transmission line

$$P_{line} = P_{line, pu} S_{base} = 0.00282 \cdot 10\,000 = 28.2\, W$$



The per-unit system

When only one device (transformer or motor) is analyzed, its own ratings are used as the basis for per-unit system. When considering a transformer in a per-unit system, transformer's characteristics will not vary much over a wide range of voltages and powers. For example, the series resistance is usually from 0.02 to 0.1 pu; the magnetizing reactance is usually from 10 to 40 pu; the core-loss resistance is usually from 50 to 200 pu. Also, the per-unit impedances of synchronous and induction machines fall within relatively narrow ranges over quite large size ranges.

If more than one transformer is present in a system, the system base voltage and power can be chosen arbitrary. However, the entire system must have the same base power, and the base voltages at various points in the system must be related by the voltage ratios of the transformers.

System base quantities are commonly chosen to the base of the largest component in the system.



The per-unit system

Per-unit values given to another base can be converted to the new base either through an intermediate step (converting them to the actual values) or directly as follows:

$$(P, Q, S)_{pu, base 2} = (P, Q, S)_{pu, base 1} \frac{S_{base 1}}{S_{base 2}} \quad (4.49.1)$$

$$V_{pu, base 2} = V_{pu, base 1} \frac{V_{base 1}}{V_{base 2}} \quad (4.49.2)$$

$$(R, X, Z)_{pu, base 2} = (R, X, Z)_{pu, base 1} \frac{V_{base 1}^2 S_{base 1}}{V_{base 2}^2 S_{base 2}} \quad (4.49.3)$$



The per-unit system: Example

Example 4.4: Sketch the appropriate per-unit equivalent circuit for the 8000/240 V, 60 Hz, 20 kVA transformer with $R_c = 159 \text{ k}\Omega$, $X_M = 38.4 \text{ k}\Omega$, $R_{eq} = 38.3 \text{ }\Omega$, $X_{eq} = 192 \text{ }\Omega$.

To convert the transformer to per-unit system, the primary circuit base impedance needs to be found.

$$V_{base1} = 8\,000 \text{ V}; \quad S_{base1} = 20\,000 \text{ VA}$$

$$Z_{base1} = \frac{V_{base1}^2}{S_{base1}} = \frac{8\,000^2}{20\,000} = 3\,200 \text{ }\Omega$$

$$Z_{SE,pu} = \frac{38.4 + j192}{3\,200} = 0.012 + j0.06 \text{ pu}$$

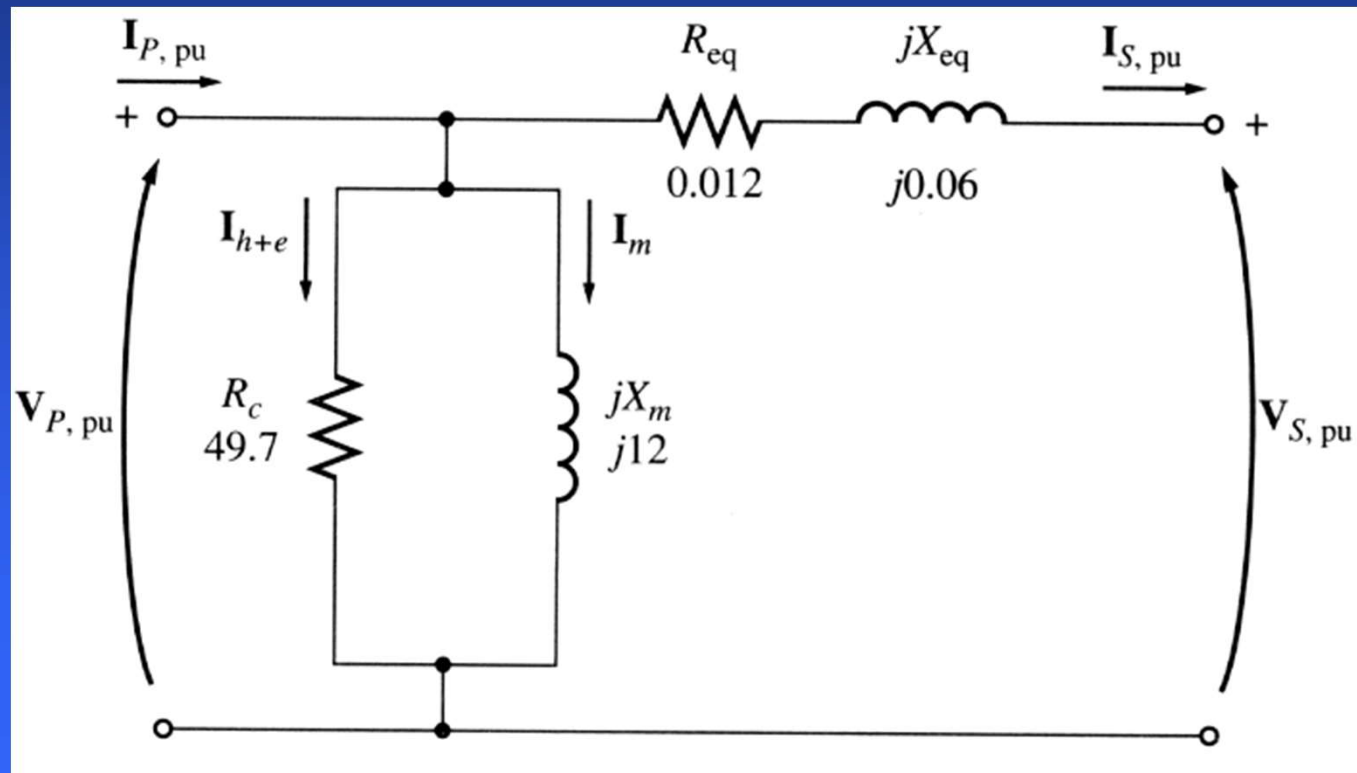
$$R_{C,pu} = \frac{159\,000}{3\,200} = 49.7 \text{ pu}$$

$$X_{M,pu} = \frac{38\,400}{3\,200} = 12 \text{ pu}$$



The per-unit system: Example

Therefore, the per-unit equivalent circuit is shown below:



Voltage regulation and efficiency

Since a real transformer contains series impedances, the transformer's output voltage varies with the load even if the input voltage is constant. To compare transformers in this respect, the quantity called a full-load voltage regulation (VR) is defined as follows:

$$VR = \frac{V_{s,nl} - V_{s,fl}}{V_{s,fl}} \cdot 100\% = \frac{V_p / a - V_{s,fl}}{V_{s,fl}} \cdot 100\% \quad (4.52.1)$$

In a per-unit system:

$$VR = \frac{V_{p,pu} - V_{s,fl,pu}}{V_{s,fl,pu}} \cdot 100\% \quad (4.52.2)$$

Where $V_{s,nl}$ and $V_{s,fl}$ are the secondary no load and full load voltages.

Note, the VR of an ideal transformer is zero.



The transformer phasor diagram

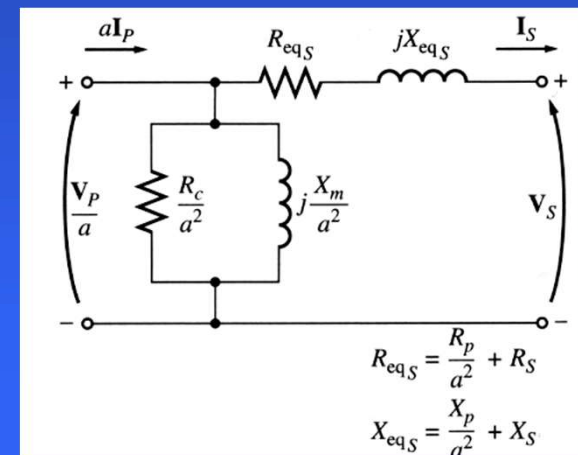
To determine the VR of a transformer, it is necessary to understand the voltage drops within it. Usually, the effects of the excitation branch on transformer VR can be ignored and, therefore, only the series impedances need to be considered. The VR depends on the magnitude of the impedances and on the current phase angle.

A phasor diagram is often used in the VR determinations. The phasor voltage V_s is assumed to be at 0° and all other voltages and currents are compared to it.

Considering the diagram and by applying the Kirchhoff's voltage law, the primary voltage is:

$$\frac{V_p}{a} = V_s + R_{eq} I_s + jX_{eq} I_s \quad (4.53.1)$$

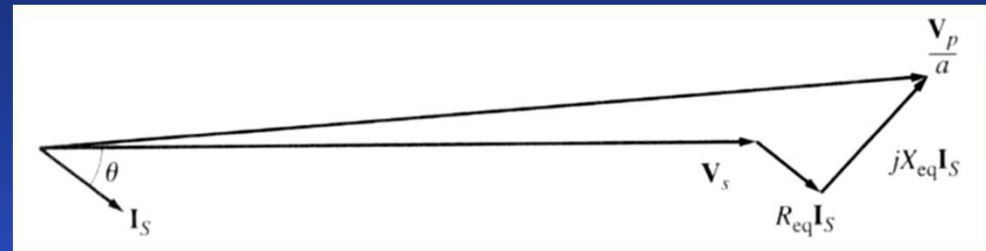
A transformer phasor diagram is a graphical representation of this equation.



The transformer phasor diagram

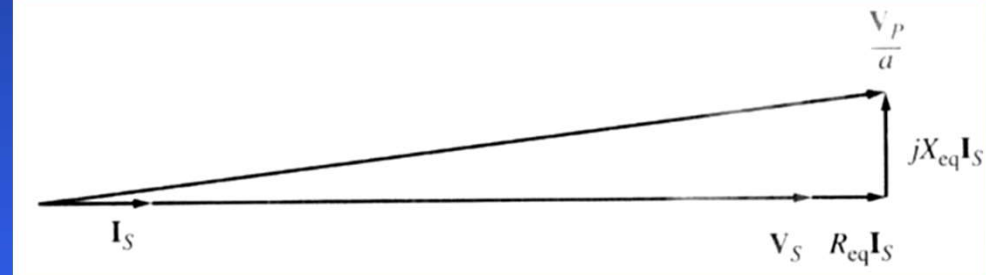
A transformer operating at a lagging power factor:

It is seen that $V_p/a > V_s$, $VR > 0$



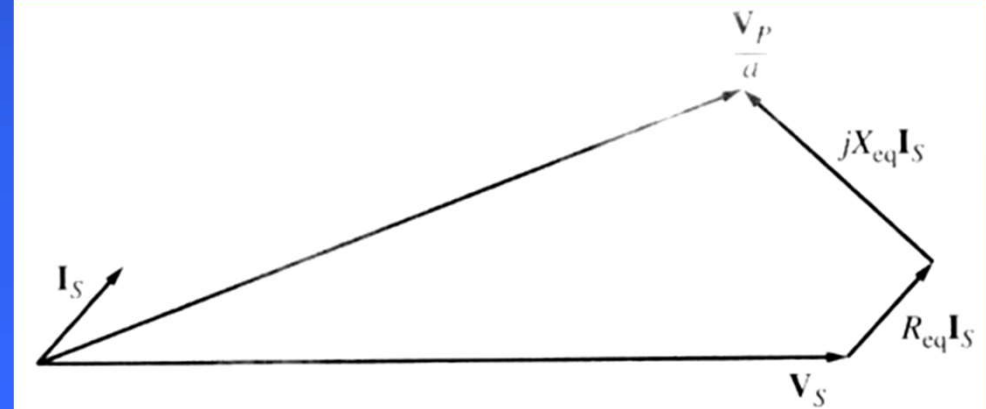
A transformer operating at a unity power factor:

It is seen that $VR > 0$



A transformer operating at a leading power factor:

If the secondary current is leading, the secondary voltage can be higher than the referred primary voltage; $VR < 0$.



The transformer efficiency

The efficiency of a transformer is defined as:

$$\eta = \frac{P_{out}}{P_{in}} \cdot 100\% = \frac{P_{out}}{P_{out} + P_{loss}} \cdot 100\% \quad (4.55.1)$$

Note: the same equation describes the efficiency of motors and generators.

Considering the transformer equivalent circuit, we notice three types of losses:

1. Copper (I^2R) losses – are accounted for by the series resistance
2. Hysteresis losses – are accounted for by the resistor R_c .
3. Eddy current losses – are accounted for by the resistor R_c .

Since the output power is
$$P_{out} = V_s I_s \cos \theta_s \quad (4.55.2)$$

The transformer efficiency is

$$\eta = \frac{V_s I_s \cos \theta}{P_{Cu} + P_{core} + V_s I_s \cos \theta} \cdot 100\% \quad (4.55.3)$$



The transformer efficiency: Example

Example 4.5: A 15 kVA, 2300/230 V transformer was tested to by open-circuit and closed-circuit tests. The following data was obtained:

$V_{OC} = 2300 \text{ V}$	$V_{SC} = 47 \text{ V}$
$I_{OC} = 0.21 \text{ A}$	$I_{SC} = 6.0 \text{ A}$
$P_{OC} = 50 \text{ W}$	$P_{SC} = 160 \text{ W}$

- Find the equivalent circuit of this transformer referred to the high-voltage side.
- Find the equivalent circuit of this transformer referred to the low-voltage side.
- Calculate the full-load voltage regulation at 0.8 lagging power factor, at 1.0 power factor, and at 0.8 leading power factor.
- Plot the voltage regulation as load is increased from no load to full load at power factors of 0.8 lagging, 1.0, and 0.8 leading.
- What is the efficiency of the transformer at full load with a power factor of 0.8 lagging?



The transformer efficiency: Example

a. The excitation branch values of the equivalent circuit can be determined as:

$$\theta_{oc} = \cos^{-1} \frac{P_{oc}}{V_{oc} I_{oc}} = \cos^{-1} \frac{50}{2300 \cdot 0.21} = 84^\circ$$

The excitation admittance is:

$$Y_E = \frac{I_{oc}}{V_{oc}} \angle -84^\circ = \frac{0.21}{2300} \angle -84^\circ = 0.000095 - j0.0000908 \text{ S}$$

The elements of the excitation branch referred to the primary side are:

$$R_c = \frac{1}{0.0000095} = 105 \text{ k}\Omega$$

$$X_M = \frac{1}{0.0000908} = 11 \text{ k}\Omega$$



The transformer efficiency: Example

From the short-circuit test data, the short-circuit impedance angle is

$$\theta_{SC} = \cos^{-1} \frac{P_{SC}}{V_{SC} I_{SC}} = \cos^{-1} \frac{160}{47 \cdot 6} = 55.4^\circ$$

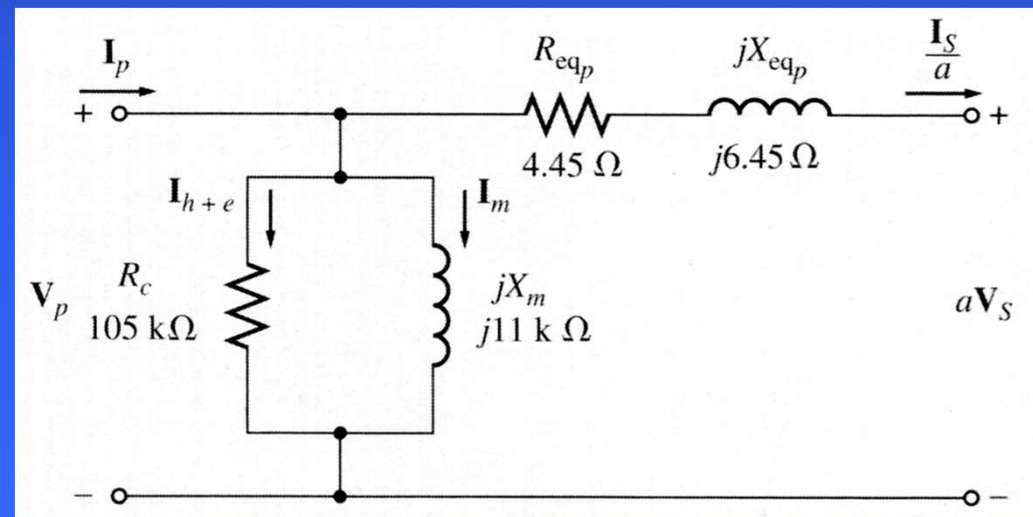
The equivalent series impedance is thus

$$Z_{SE} = \frac{V_{SC}}{I_{SC}} \angle \theta_{SC} = \frac{47}{6} \angle 55.4^\circ = 4.45 + j6.45 \Omega$$

The series elements referred to the primary winding are:

$$R_{eq} = 4.45 \Omega; \quad X_{eq} = 6.45 \Omega$$

The equivalent circuit

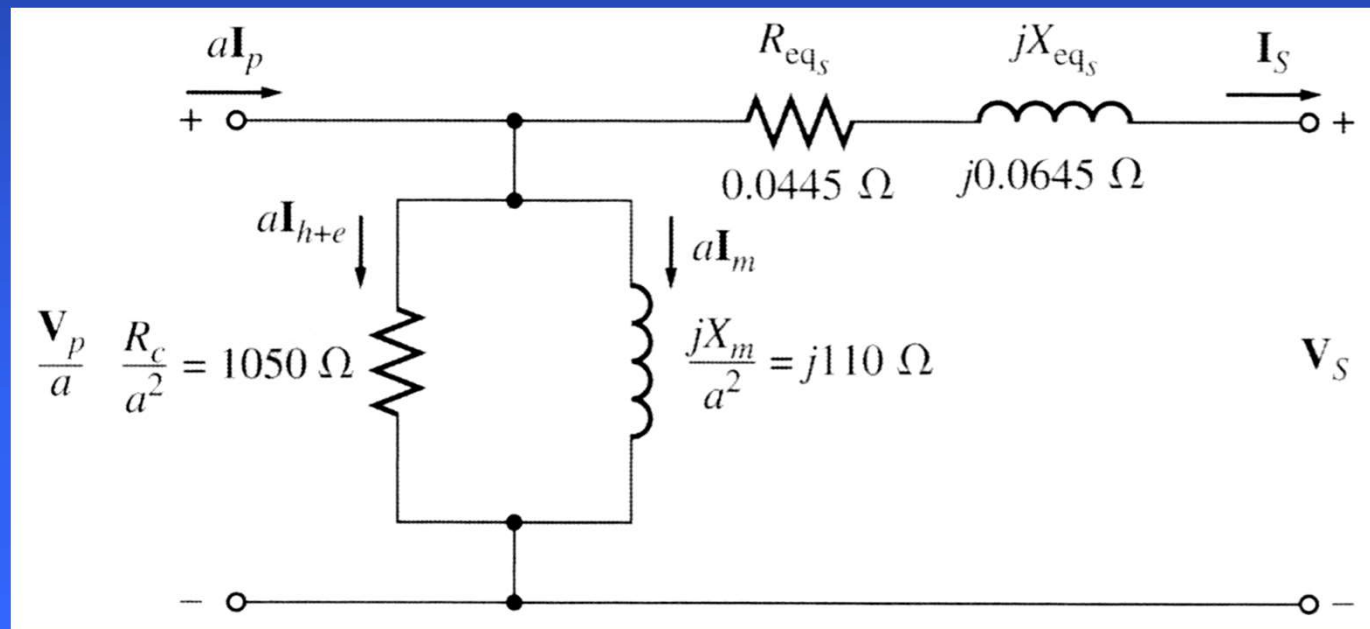


The transformer efficiency: Example

b. To find the equivalent circuit referred to the low-voltage side, we need to divide the impedance by a^2 . Since $a = 10$, the values will be:

$$R_C = 1050 \Omega \quad X_M = 110 \Omega \quad R_{eq} = 0.0445 \Omega \quad X_{eq} = 0.0645 \Omega$$

The equivalent circuit will be



The transformer efficiency: Example

c. The full-load current on the secondary side of the transformer is

$$I_{S, rated} = \frac{S_{rated}}{V_{S, rated}} = \frac{15\,000}{230} = 65.2 \text{ A}$$

Since:

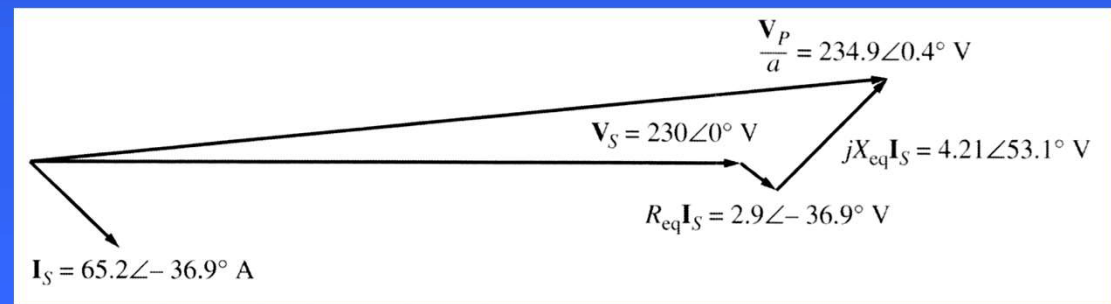
$$\frac{V_p}{a} = V_S + R_{eq} I_S + jX_{eq} I_S$$

At PF = 0.8 lagging, current $I_S = 65.2 \angle -\cos^{-1}(0.8) = 65.2 \angle -36.9^\circ \text{ A}$

and $\frac{V_p}{a} = 230 \angle 0^\circ + 0.0445 \cdot (65.2 \angle -36.9^\circ) + j0.0645 \cdot (65.2 \angle -36.9^\circ) = 234.85 \angle 0.40^\circ \text{ V}$

The resulting voltage regulation is, therefore:

$$\begin{aligned} VR &= \frac{|V_p/a| - V_{S, fl}}{V_{S, fl}} \cdot 100\% \\ &= \frac{234.85 - 230}{230} \cdot 100\% \\ &= 2.1\% \end{aligned}$$



The transformer efficiency: Example

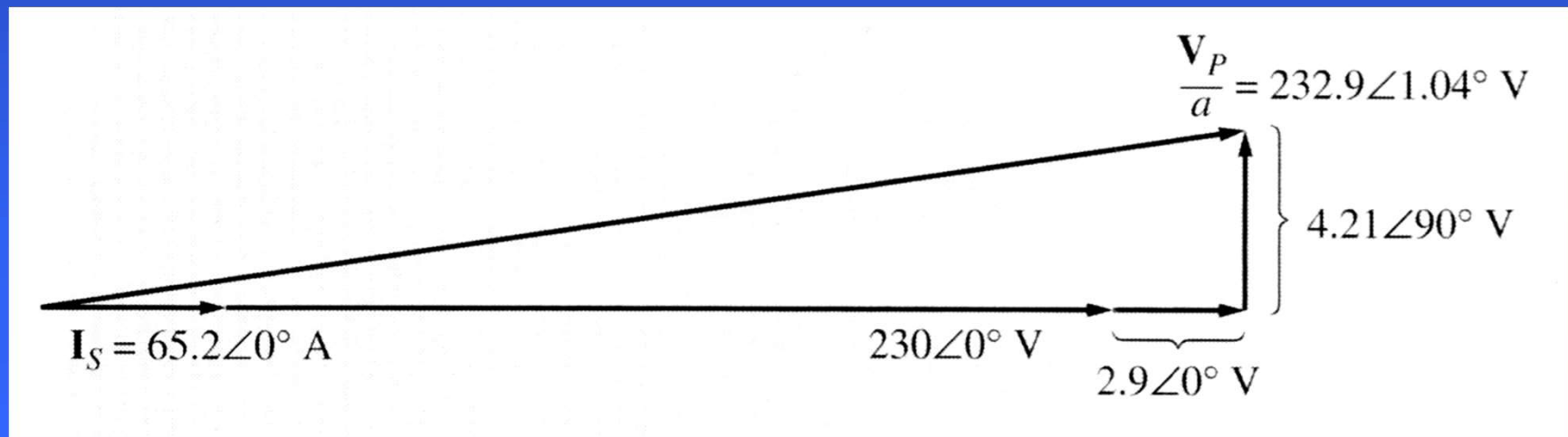
At PF = 1.0, current

$$I_s = 65.2 \angle \cos^{-1}(1.0) = 65.2 \angle 0^\circ \text{ A}$$

and
$$\frac{V_p}{a} = 230 \angle 0^\circ + 0.0445 \cdot (65.2 \angle 0^\circ) + j0.0645 \cdot (65.2 \angle 0^\circ) = 232.94 \angle 1.04^\circ \text{ V}$$

The resulting voltage regulation is, therefore:

$$VR = \frac{|V_p/a| - V_{S,fl}}{V_{S,fl}} \cdot 100\% = \frac{232.94 - 230}{230} \cdot 100\% = 1.28\%$$



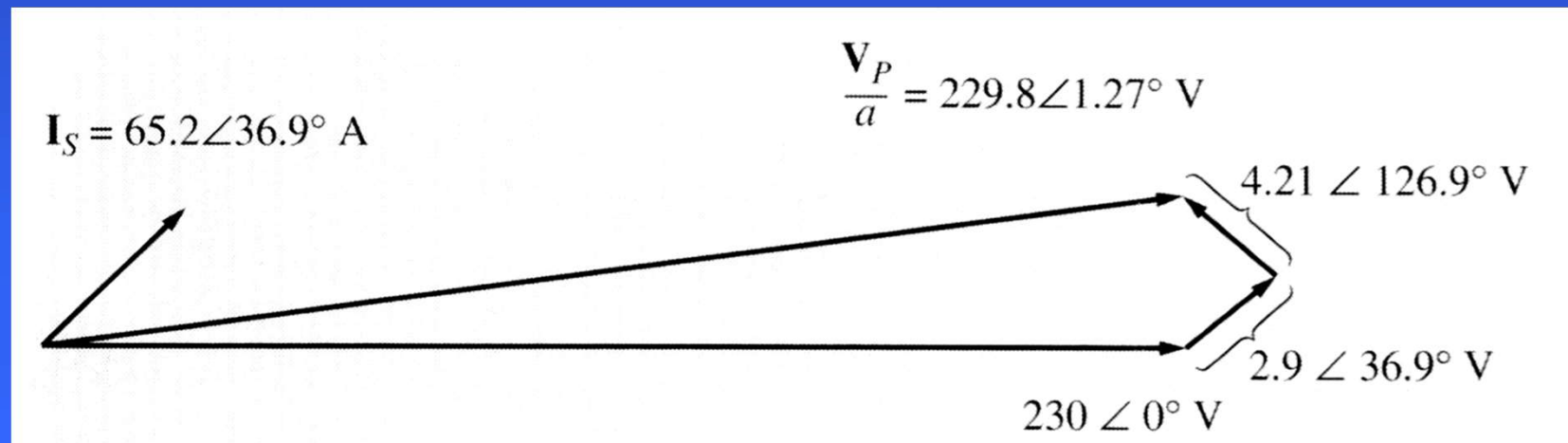
The transformer efficiency: Example

At PF = 0.8 leading, current $I_s = 65.2 \angle \cos^{-1}(0.8) = 65.2 \angle 36.9^\circ \text{ A}$

and $\frac{V_p}{a} = 230 \angle 0^\circ + 0.0445 \cdot (65.2 \angle 36.9^\circ) + j0.0645 \cdot (65.2 \angle 36.9^\circ) = 229.85 \angle 1.27^\circ \text{ V}$

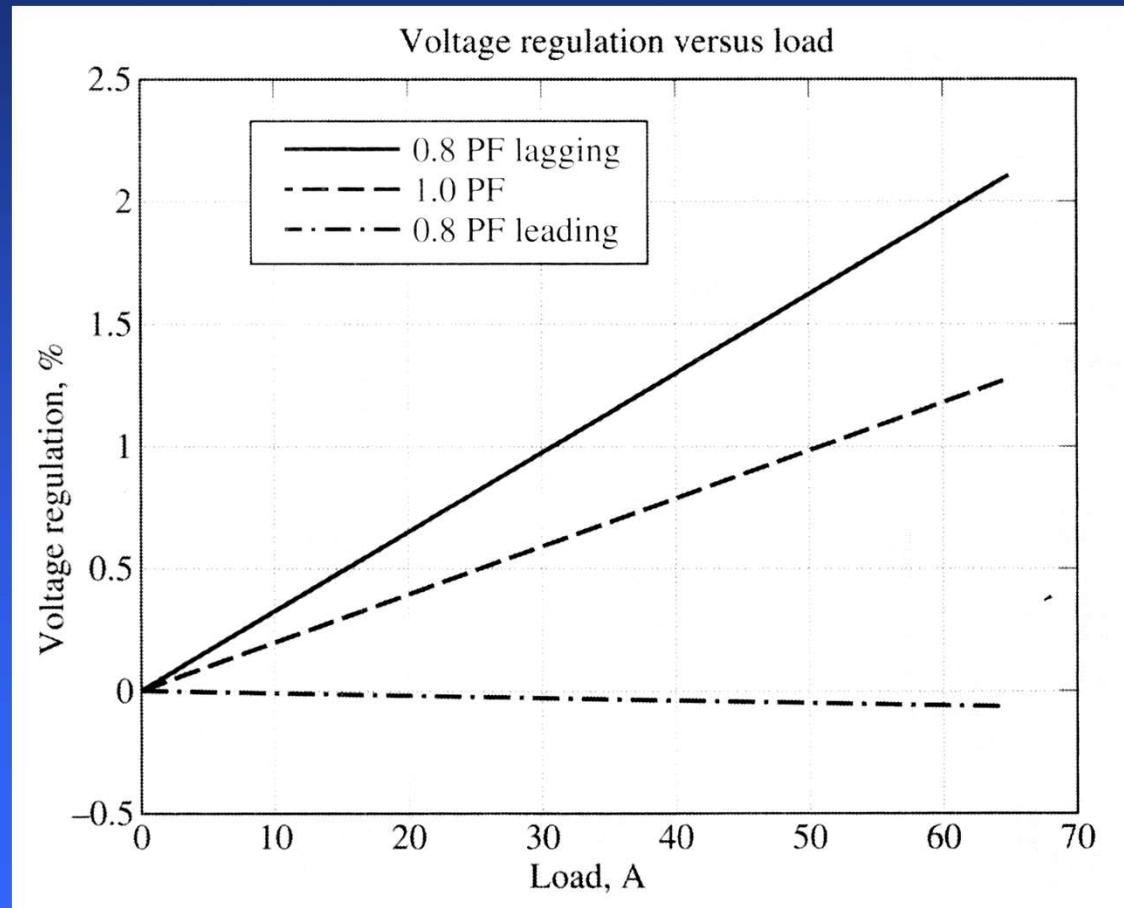
The resulting voltage regulation is, therefore:

$$VR = \frac{|V_p/a| - V_{S,fl}}{V_{S,fl}} \cdot 100\% = \frac{229.85 - 230}{230} \cdot 100\% = -0.062\%$$



The transformer efficiency: Example

Similar computations can be repeated for different values of load current. As a result, we can plot the voltage regulation as a function of load current for the three Power Factors.



The transformer efficiency: Example

e. To find the efficiency of the transformer, first calculate its losses.

The copper losses are:

$$P_{Cu} = I_S^2 R_{eq} = 65.2^2 \cdot 0.0445 = 189 \text{ W}$$

The core losses are:

$$P_{core} = \frac{(V_p/a)^2}{R_C} = \frac{234.85^2}{1050} = 52.5 \text{ W}$$

The output power of the transformer at the given Power Factor is:

$$P_{out} = V_S I_S \cos \theta = 230 \cdot 65.2 \cdot \cos 36.9^\circ = 12\,000 \text{ W}$$

Therefore, the efficiency of the transformer is

$$\eta = \frac{P_{out}}{P_{Cu} + P_{core} + P_{out}} \cdot 100\% = 98.03\%$$



Transformer taps and voltage regulation

We assumed before that the transformer turns ratio is a fixed (constant) for the given transformer. Frequently, distribution transformers have a series of taps in the windings to permit small changes in their turns ratio. Typically, transformers may have 4 taps in addition to the nominal setting with spacing of 2.5 % of full-load voltage. Therefore, adjustments up to 5 % above or below the nominal voltage rating of the transformer are possible.

Example 4.6: A 500 kVA, 13 200/480 V transformer has four 2.5 % taps on its primary winding. What are the transformer's voltage ratios at each tap setting?

+ 5.0% tap	13 860/480 V
+ 2.5% tap	13 530/480 V
Nominal rating	13 200/480 V
- 2.5% tap	12 870/480 V
- 5.0% tap	12 540/480 V



Transformer taps and voltage regulation

Taps allow adjustment of the transformer in the field to accommodate for local voltage variations.

Sometimes, transformers are used on a power line, whose voltage varies widely with the load (due to high line impedance, for instance). Normal loads need fairly constant input voltage though...

One possible solution to this problem is to use a special transformer called a tap changing under load (TCUL) transformer or voltage regulator. TCUL is a transformer with the ability to change taps while power is connected to it. A voltage regulator is a TCUL with build-in voltage sensing circuitry that automatically changes taps to keep the system voltage constant.

These “self-adjusting” transformers are very common in modern power systems.

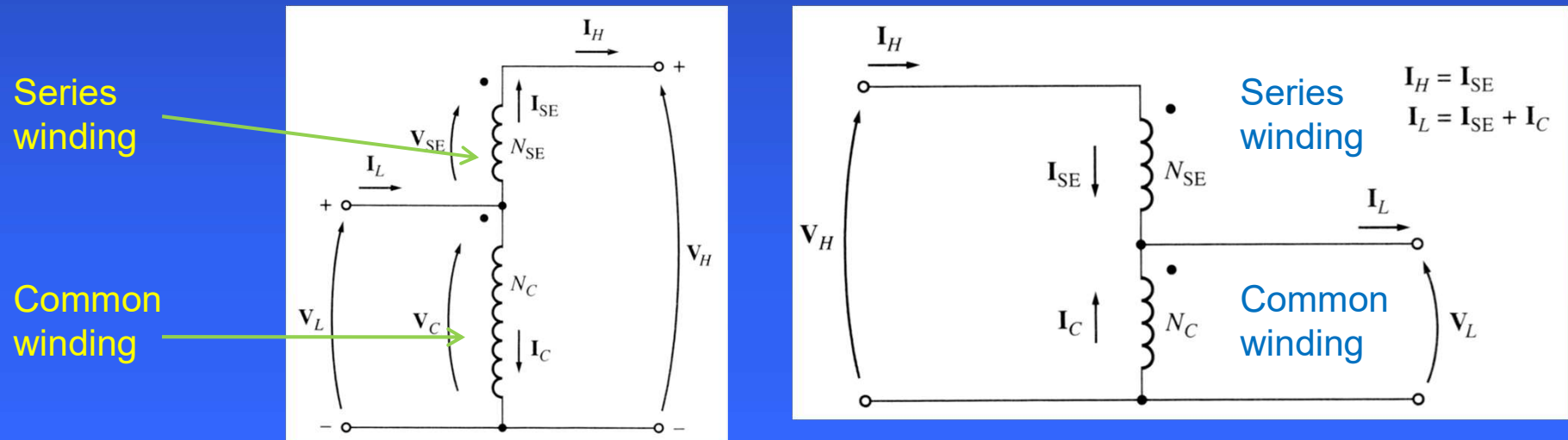


The autotransformer

Sometimes, it is desirable to change the voltage by a small amount (for instance, when the consumer is far away from the generator and it is needed to raise the voltage to compensate for voltage drops).

In such situations, it would be expensive to wind a transformer with two windings of approximately equal number of turns. An autotransformer (a transformer with only one winding) is used instead.

Diagrams of step-up and step-down autotransformers:



Output (up) or input (down) voltage is a sum of voltages across common and series windings.



The autotransformer

Since the autotransformer's coils are physically connected, a different terminology is used for autotransformers:

The voltage across the common winding is called a common voltage V_C , and the current through this coil is called a common current I_C . The voltage across the series winding is called a series voltage V_{SE} , and the current through that coil is called a series current I_{SE} .

The voltage and current on the low-voltage side are called V_L and I_L ; the voltage and current on the high-voltage side are called V_H and I_H .

For the autotransformers:

$$\frac{V_C}{V_{SE}} = \frac{N_C}{N_{SE}} \quad (4.68.1)$$

$$N_C I_C = N_{SE} I_{SE} \quad (4.68.2)$$

$$V_L = V_C \quad I_L = I_C + I_{SE} \quad (4.68.3)$$

$$V_H = V_C + V_{SE} \quad I_H = I_{SE} \quad (4.68.4)$$



Voltage and Current relationships in an Autotransformer

Combining (4.68.1) through (4.68.4), for the high-side voltage, we arrive at

$$V_H = V_C + \frac{N_{SE}}{N_C} V_C = V_L + \frac{N_{SE}}{N_C} V_L \quad (4.69.1)$$

Therefore:

$$\frac{V_L}{V_H} = \frac{N_C}{N_C + N_{SE}} \quad (4.69.2)$$

The current relationship will be:

$$I_L = I_{SE} + \frac{N_{SE}}{N_C} I_{SE} = I_H + \frac{N_{SE}}{N_C} I_H \quad (4.69.3)$$

Therefore:

$$\frac{I_L}{I_H} = \frac{N_C + N_{SE}}{N_C} \quad (4.69.4)$$



The apparent power advantage

Not all the power traveling from the primary to the secondary winding of the autotransformer goes through the windings. As a result, an autotransformer can handle much power than the conventional transformer (with the same windings).

Considering a step-up autotransformer, the apparent input and output powers are:

$$S_{in} = V_L I_L \quad (4.70.1)$$

$$S_{out} = V_H I_H \quad (4.70.2)$$

It is easy to show that $S_{in} = S_{out} = S_{IO}$ (4.70.3)

where S_{IO} is the input and output apparent powers of the autotransformer. However, the apparent power in the autotransformer's winding is

$$S_W = V_C I_C = V_{SE} I_{SE} \quad (4.70.4)$$

Which is:

$$\begin{aligned} S_W &= V_L (I_L - I_H) = V_L I_L - V_L I_H \\ &= V_L I_L - V_L I_L \frac{N_C}{N_{SE} + N_C} = S_{IO} \frac{N_{SE}}{N_{SE} + N_C} \end{aligned} \quad (4.70.5)$$



The apparent power advantage

Therefore, the ratio of the apparent power in the primary and secondary of the autotransformer to the apparent power **actually** traveling through its windings is

$$\frac{S_{IO}}{S_W} = \frac{N_{SE} + N_C}{N_{SE}} \quad (4.71.1)$$

The last equation described the apparent power rating advantage of an autotransformer over a conventional transformer.

S_W is the apparent power actually passing through the windings. The rest passes from primary to secondary parts without being coupled through the windings.

Note that the smaller the series winding, the greater the advantage!



The apparent power advantage

For example, a 5 MVA autotransformer that connects a 110 kV system to a 138 kV system would have a turns ratio (common to series) 110:28. Such an autotransformer would actually have windings rated at:

$$S_W = S_{IO} \frac{N_{SE}}{N_{SE} + N_C} = 5 \cdot \frac{28}{28 + 110} = 1.015 \text{ MVA}$$

Therefore, the autotransformer would have windings rated at slightly over 1 MVA instead of 5 MVA, which makes it 5 times smaller and, therefore, considerably less expensive.

However, the construction of autotransformers is usually slightly different. In particular, the insulation on the smaller coil (the series winding) of the autotransformer is made as strong as the insulation on the larger coil to withstand the full output voltage.

The primary disadvantage of an autotransformer is that there is a direct physical connection between its primary and secondary circuits. Therefore, the electrical isolation of two sides is lost.



The apparent power advantage: Ex

Example 4.7: A 100 VA, 120/12 V transformer will be connected to form a step-up autotransformer with the primary voltage of 120 V.

- What will be the secondary voltage?
- What will be the maximum power rating?
- What will be the power rating advantage?

a. The secondary voltage:

$$V_H = \frac{N_C + N_{SE}}{N_C} V_L = \frac{120 + 12}{120} 120 = 132 \text{ V}$$

b. The max series winding current:

$$I_{SE, \max} = \frac{S_{\max}}{V_{SE}} = \frac{100}{12} = 8.33 \text{ A}$$

The secondary apparent power:

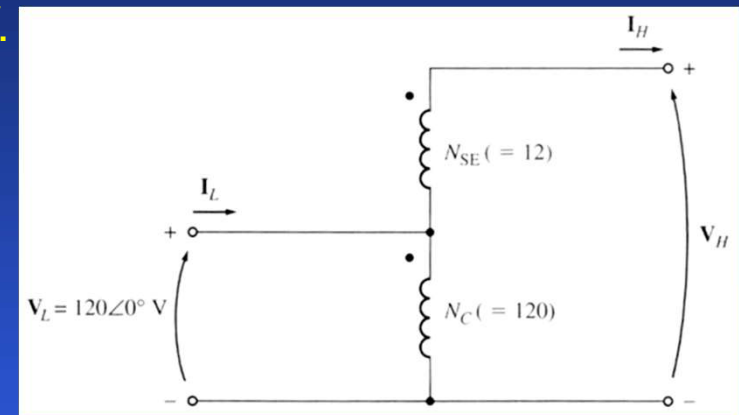
$$S_{out} = V_S I_S = V_H I_H = 132 \cdot 8.33 = 1100 \text{ VA}$$

c. The power rating advantage:

$$\frac{S_{IO}}{S_W} = \frac{1100}{100} = 11$$

or

$$\frac{S_{IO}}{S_W} = \frac{N_{SE} + N_C}{N_{SE}} = \frac{120 + 12}{12} = \frac{132}{12} = 11$$



Variable-voltage autotransformers

The effective per-unit impedance of an autotransformer is smaller than of a conventional transformer by a reciprocal to its power advantage. This is an additional disadvantage of autotransformers.

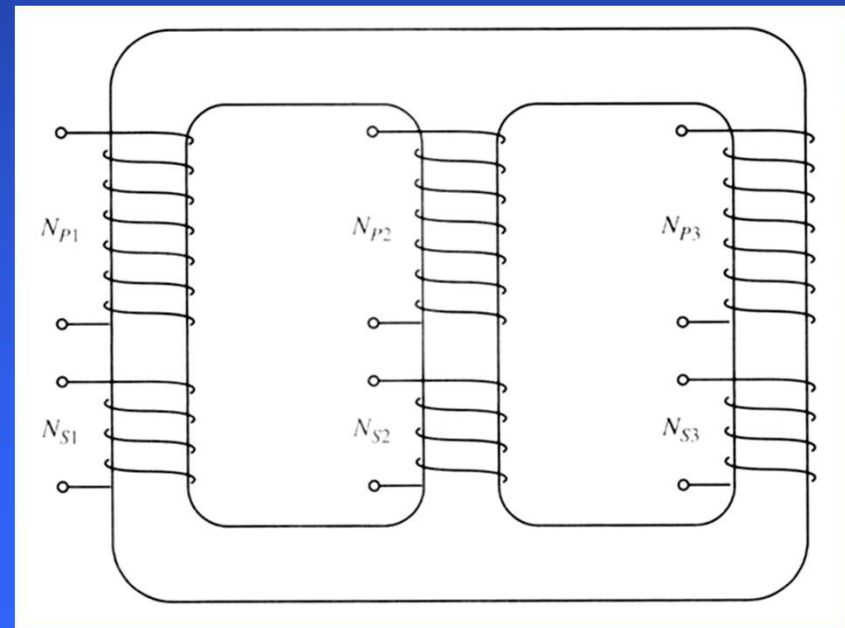
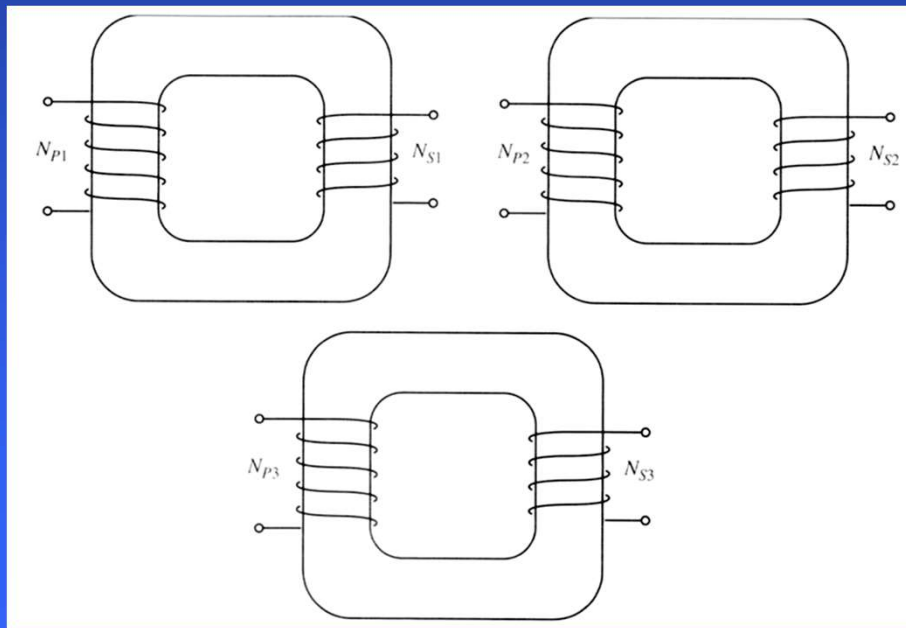


It is a common practice to make variable voltage autotransformers.



3-phase transformers

The majority of the power generation/distribution systems in the world are 3-phase systems. The transformers for such circuits can be constructed either as a 3-phase bank of independent identical transformers (can be replaced independently) or as a single transformer wound on a single 3-legged core (lighter, cheaper, more efficient).



3-phase transformer connections

We assume that any single transformer in a 3-phase transformer (bank) behaves exactly as a single-phase transformer. The impedance, voltage regulation, efficiency, and other calculations for 3-phase transformers are done on a per-phase basis, using the techniques studied previously for single-phase transformers.

Four possible connections for a 3-phase transformer bank are:

1. Y-Y
2. Y- Δ
3. Δ - Δ
4. Δ -Y



3-phase transformer connections

1. Y-Y connection:

The primary voltage on each phase of the transformer is

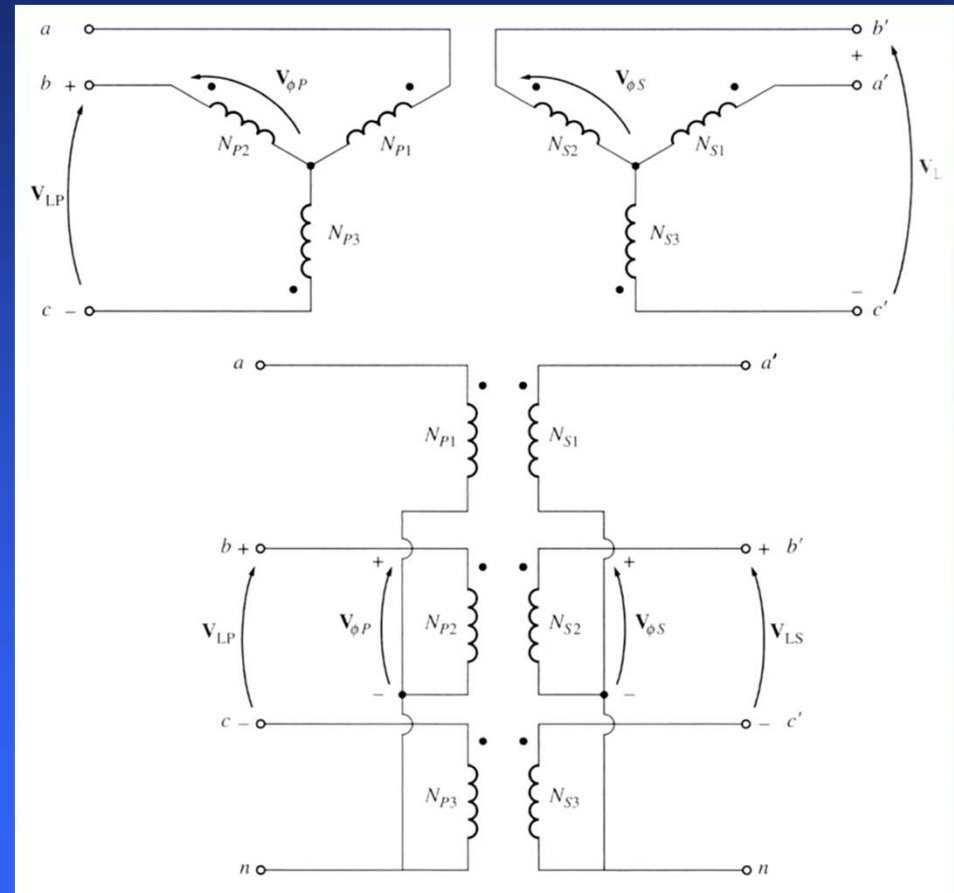
$$V_{\phi P} = \frac{V_{LP}}{\sqrt{3}} \quad (4.77.1)$$

The secondary phase voltage is

$$V_{LS} = \sqrt{3}V_{\phi S} \quad (4.77.2)$$

The overall voltage ratio is

$$\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}V_{\phi P}}{\sqrt{3}V_{\phi S}} = a \quad (4.77.3)$$



3-phase transformer connections

The Y-Y connection has two very serious problems:

1. If loads on one of the transformer circuits are unbalanced, the voltages on the phases of the transformer can become severely unbalanced.
2. The third harmonic issue. The voltages in any phase of an Y-Y transformer are 120° apart from the voltages in any other phase. However, the third-harmonic components of each phase will be in phase with each other. Nonlinearities in the transformer core always lead to generation of third harmonic! These components will add up resulting in large (can be even larger than the fundamental component) third harmonic component.

Both problems can be solved by one of two techniques:

1. Solidly ground the neutral of the transformers (especially, the primary side). The third harmonic will flow in the neutral and a return path will be established for the unbalanced loads.
2. Add a third Δ -connected winding. A circulating current at the third harmonic will flow through it suppressing the third harmonic in other windings.



3-phase transformer connections

2. Y-Δ connection:

The primary voltage on each phase of the transformer is

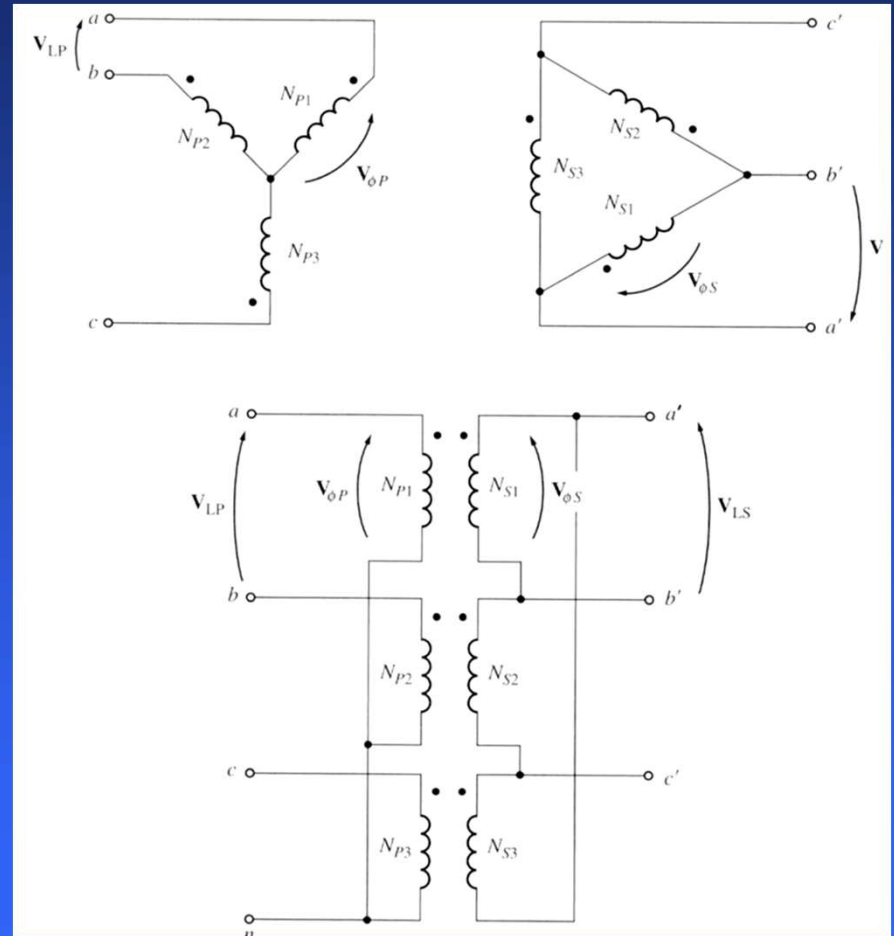
$$V_{\phi P} = \frac{V_{LP}}{\sqrt{3}} \quad (4.79.1)$$

The secondary phase voltage is

$$V_{LS} = V_{\phi S} \quad (4.79.2)$$

The overall voltage ratio is

$$\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}V_{\phi P}}{V_{\phi S}} = \sqrt{3}a \quad (4.79.3)$$



3-phase transformer connections

The Y- Δ connection has no problem with third harmonic components due to circulating currents in Δ . It is also more stable to unbalanced loads since the Δ partially redistributes any imbalance that occurs.

One problem associated with this connection is that the secondary voltage is shifted by 30° with respect to the primary voltage. This can cause problems when paralleling 3-phase transformers since transformers secondary voltages must be in-phase to be paralleled. Therefore, we must pay attention to these shifts.

In the U.S., it is common to make the secondary voltage to lag the primary voltage. The connection shown in the previous slide will do it.



3-phase transformer connections

3. Δ -Y connection:

The primary voltage on each phase of the transformer is

$$V_{\phi P} = V_{LP} \quad (4.81.1)$$

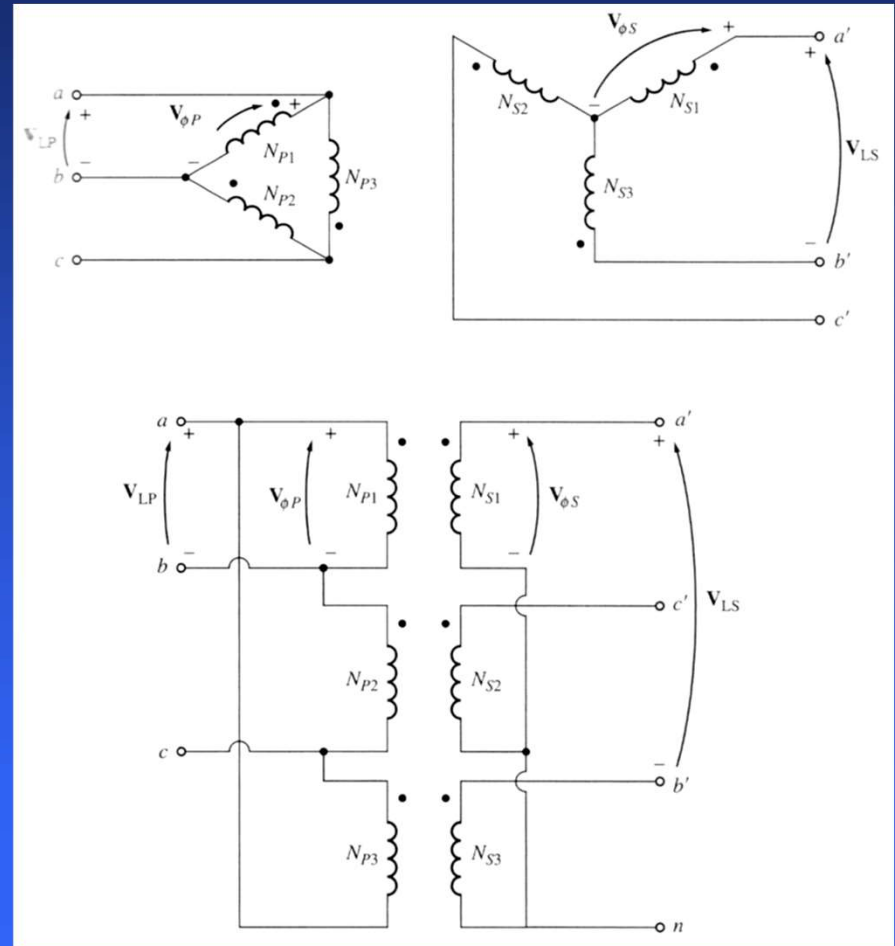
The secondary phase voltage is

$$V_{LS} = \sqrt{3}V_{\phi S} \quad (4.81.2)$$

The overall voltage ratio is

$$\frac{V_{LP}}{V_{LS}} = \frac{V_{\phi P}}{\sqrt{3}V_{\phi S}} = \frac{a}{\sqrt{3}} \quad (4.81.3)$$

The same advantages and the same phase shift as the Y- Δ connection.



3-phase transformer connections

4. $\Delta - \Delta$ connection:

The primary voltage on each phase of the transformer is

$$V_{\phi P} = V_{LP} \quad (4.82.1)$$

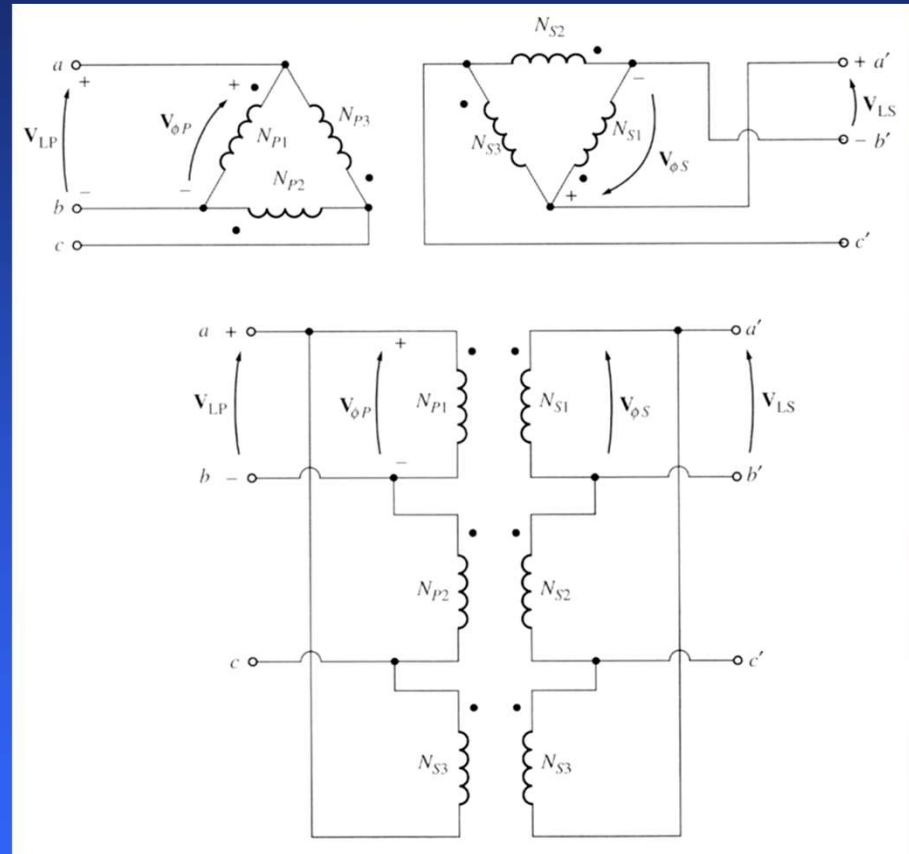
The secondary phase voltage is

$$V_{LS} = V_{\phi S} \quad (4.82.2)$$

The overall voltage ratio is

$$\frac{V_{LP}}{V_{LS}} = \frac{V_{\phi P}}{V_{\phi S}} = a \quad (4.82.3)$$

No phase shift, no problems with unbalanced loads or harmonics.



3-phase transformer: per-unit system

The per-unit system applies to the 3-phase transformers as well as to single-phase transformers. If the total base VA value of the transformer bank is S_{base} , the base VA value of one of the transformers will be

$$S_{1\phi,base} = \frac{S_{base}}{3} \quad (4.83.1)$$

Therefore, the base phase current and impedance of the transformer are

$$I_{\phi,base} = \frac{S_{1\phi,base}}{V_{\phi,base}} = \frac{S_{base}}{3V_{\phi,base}} \quad (4.83.2)$$

$$Z_{base} = \frac{(V_{\phi,base})^2}{S_{1\phi,base}} = \frac{3(V_{\phi,base})^2}{S_{base}} \quad (4.83.3)$$



3-phase transformer: per-unit system

The line quantities on 3-phase transformer banks can also be represented in per-unit system. If the windings are in Δ :

$$V_{L,base} = V_{\phi,base} \quad (4.84.1)$$

If the windings are in Y:

$$V_{L,base} = \sqrt{3}V_{\phi,base} \quad (4.84.2)$$

And the base line current in a 3-phase transformer bank is

$$I_{L,base} = \frac{S_{base}}{\sqrt{3}V_{L,base}} \quad (4.84.3)$$

The application of the per-unit system to 3-phase transformer problems is similar to its application in single-phase situations. The voltage regulation of the transformer bank is the same.



3-phase transformer: per-unit system: Ex

Example 4.8: A 50 kVA, 13 800/208 V Δ -Y transformer has a resistance of 1% and a reactance of 7% per unit.

- What is the transformer's phase impedance referred to the high voltage side?
- What is the transformer's voltage regulation at full load and 0.8 PF lagging, using the calculated high-side impedance?
- What is the transformer's voltage regulation under the same conditions, using the per-unit system?

a. The high-voltage side of the transformer has the base voltage 13 800 V and a base apparent power of 50 kVA. Since the primary side is Δ -connected, its phase voltage and the line voltage are the same. The base impedance is:

$$Z_{base} = \frac{3(V_{\phi,base})^2}{S_{base}} = \frac{3(13\,800)^2}{50\,000} = 11\,426 \Omega$$



3-phase transformer: per-unit system: Ex

The per-unit impedance of the transformer is:

$$Z_{eq,pu} = 0.01 + j0.07 \text{ pu}$$

Therefore, the high-side impedance in ohms is:

$$Z_{eq} = Z_{eq,pu} Z_{base} = (0.01 + j0.07) \cdot 11426 = 114 + j800 \Omega$$

b. The voltage regulation of a 3-phase transformer equals to a voltage regulation of a single transformer:

$$VR = \frac{V_{\phi P} - aV_{\phi S}}{aV_{\phi S}} \cdot 100\%$$

The rated phase current on the primary side can be found as:

$$I_{\phi} = \frac{S}{3V_{\phi}} = \frac{50\,000}{3 \cdot 13\,800} = 1.208 \text{ A}$$



3-phase transformer: per-unit system: Ex

The rated phase voltage on the secondary of the transformer is

$$V_{\phi S} = \frac{208}{\sqrt{3}} = 120 \text{ V}$$

When referred to the primary (high-voltage) side, this voltage becomes

$$V_{\phi S}' = aV_{\phi S} = 13\,800 \text{ V}$$

Assuming that the transformer secondary winding is working at the rated voltage and current, the resulting primary phase voltage is

$$\begin{aligned} V_{\phi P} &= aV_{\phi S} + R_{eq}I_{\phi} + jX_{eq}I_{\phi} = 13\,800\angle 0^{\circ} + 114.2 \cdot 1.208\angle \cos^{-1}(-0.8) + j \cdot 800 \cdot 1.208\angle \cos^{-1}(-0.8) \\ &= 14\,490 + j690.3 = 14\,506\angle 2.73^{\circ} \text{ V} \end{aligned}$$

The voltage regulation, therefore, is

$$VR = \frac{|V_{\phi P}| - aV_{\phi S}}{aV_{\phi S}} \cdot 100\% = \frac{14\,506 - 13\,800}{13\,800} \cdot 100\% = 5.1\%$$



3-phase transformer: per-unit system: Ex

c. In the per-unit system, the output voltage is $1\angle 0^\circ$, and the current is $1\angle \cos^{-1}(-0.8)$. Therefore, the input voltage is

$$V_{\phi P} = 1\angle 0^\circ + 0.01 \cdot 1\angle \cos^{-1}(0.8) + j0.07 \cdot 1\angle \cos^{-1}(0.8) = 1.051\angle 2.73^\circ$$

Thus, the voltage regulation in per-unit system will be

$$VR = \frac{1.051 - 1.0}{1.0} \cdot 100\% = 5.1\%$$

The voltage regulation in per-unit system is the same as computed in volts...



Transformer ratings

Transformers have the following major ratings:

1. Apparent power;
2. Voltage;
3. Current;
4. Frequency.



Transformer ratings: Voltage and Frequency

The **voltage** rating is a) used to protect the winding insulation from breakdown; b) related to the magnetization current of the transformer (more important)

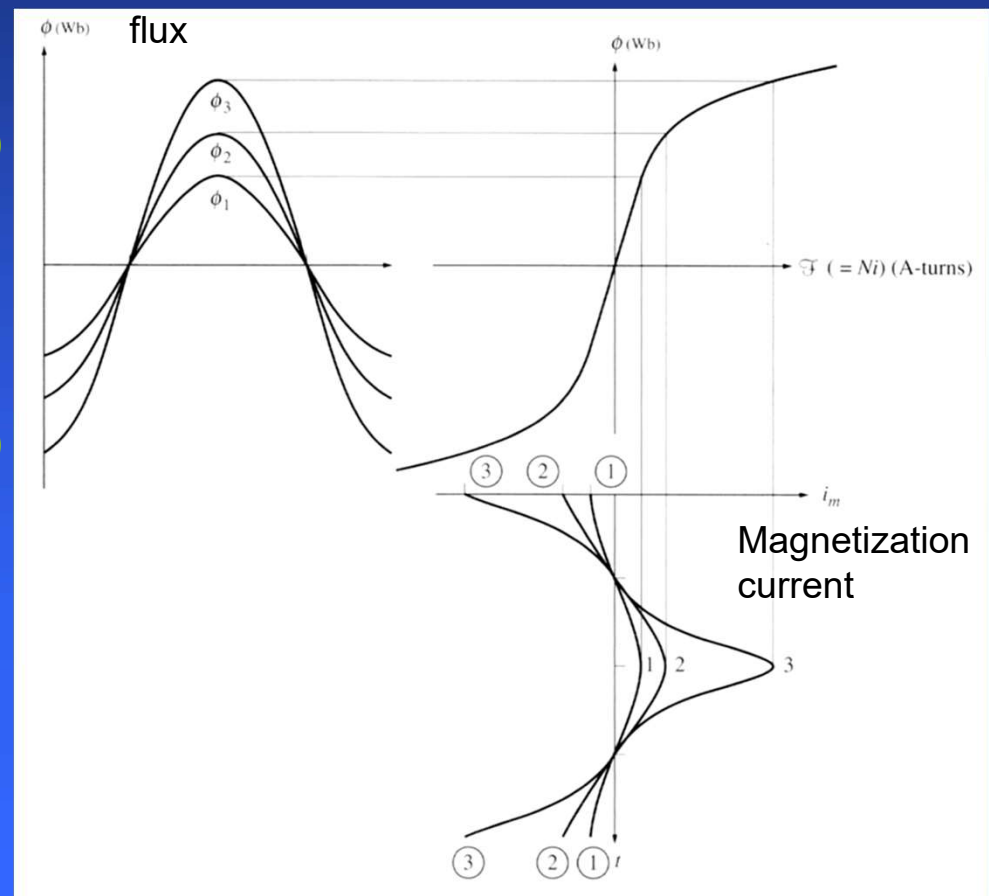
If a steady-state voltage

$$v(t) = V_M \sin \omega t \quad (4.90.1)$$

is applied to the transformer's primary winding, the transformer's flux will be

$$\phi(t) = \frac{1}{N_p} \int v(t) dt = -\frac{V_M}{\omega N_p} \cos \omega t \quad (4.90.2)$$

An increase in voltage will lead to a proportional increase in flux. However, after some point (in a saturation region), such increase in flux would require an unacceptable increase in magnetization current!



Transformer ratings: Voltage and Frequency

Therefore, the maximum applied voltage (and thus the rated voltage) is set by the maximum acceptable magnetization current in the core.

We notice that the maximum flux is also related to the frequency:

$$\phi_{\max} = \frac{V_{\max}}{\omega N_p}$$

(4.91.1)

Therefore, to maintain the same maximum flux, a change in frequency (say, 50 Hz instead of 60 Hz) must be accompanied by the corresponding correction in the maximum allowed voltage. This reduction in applied voltage with frequency is called derating. As a result, a 50 Hz transformer may be operated at a 20% higher voltage on 60 Hz if this would not cause insulation damage.



Transformer ratings: Apparent Power

The apparent power rating sets (together with the voltage rating) the current through the windings. The current determines the i^2R losses and, therefore, the heating of the coils. Remember, overheating shortens the life of transformer's insulation!

In addition to apparent power rating for the transformer itself, additional (higher) rating(s) may be specified if a forced cooling is used. Under any circumstances, the temperature of the windings must be limited.

Note, that if the transformer's voltage is reduced (for instance, the transformer is working at a lower frequency), the apparent power rating must be reduced by an equal amount to maintain the constant current.



Transformer ratings: Current inrush

Assuming that the following voltage is applied to the transformer at the moment it is connected to the line:

$$v(t) = V_M \sin(\omega t + \theta) \quad (4.93.1)$$

The maximum flux reached on the first half-cycle depends on the phase of the voltage at the instant the voltage is applied. If the initial voltage is

$$v(t) = V_M \sin(\omega t + 90^\circ) = V_M \cos \omega t \quad (4.93.2)$$

and the initial flux in the core is zero, the maximum flux during the first half-cycle is equal to the maximum steady-state flux (which is ok):

$$\phi_{\max} = \frac{V_M}{\omega N_p} \quad (4.93.3)$$

However, if the voltage's initial phase is zero, i.e.

$$v(t) = V_M \sin(\omega t) \quad (4.93.4)$$



Transformer ratings: Current inrush

the maximum flux during the first half-cycle will be

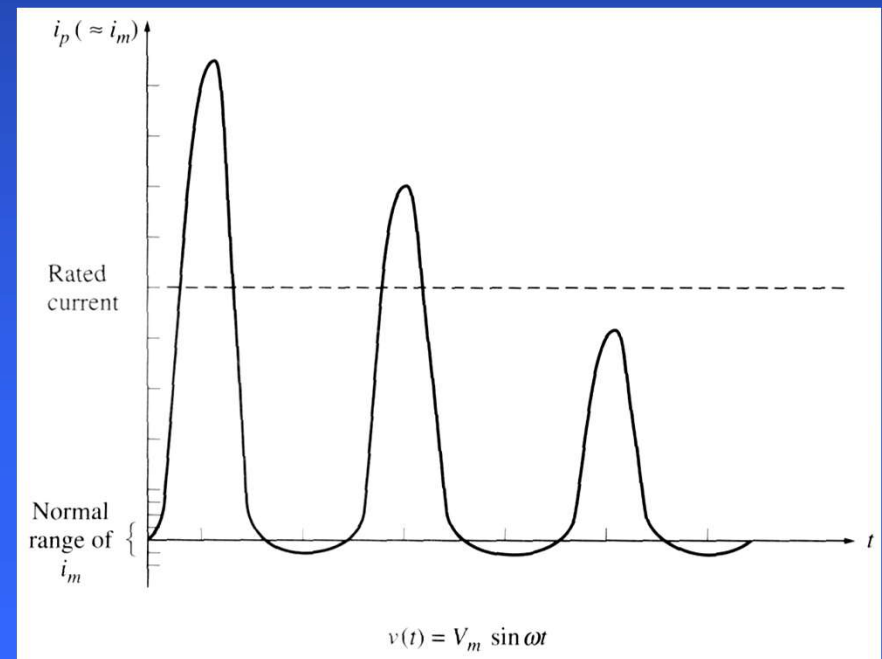
$$\phi_{\max} = \frac{1}{N_p} \int_0^{\pi/\omega} V_M \sin(\omega t) dt = -\frac{V_M}{\omega N_p} \cos(\omega t) \Big|_0^{\pi/\omega} = \frac{2V_M}{\omega N_p} \quad (4.94.1)$$

Which is twice higher than a normal steady-state flux!

Doubling the maximum flux in the core can bring the core in a saturation and, therefore, may result in a huge magnetization current!

Normally, the voltage phase angle cannot be controlled. As a result, a large inrush current is possible during the first several cycles after the transformer is turned ON.

The transformer and the power system must be able to handle these currents.



Instrument transformers

Two special-purpose transformers are used to take measurements: potential and current transformers.

A potential transformer has a high-voltage primary, low-voltage secondary, and very low power rating. It is used to provide an accurate voltage sample to instruments monitoring the power system.

A current transformer samples the current in a line and reduces it to a safe and measurable level. Such transformer consists of a secondary winding wrapped around a ferromagnetic ring with a single primary line (that may carry a large current) running through its center. The ring holds a small sample of the flux from the primary line. That flux induces a secondary voltage.

Windings in current transformers are loosely coupled: the mutual flux is much smaller than the leakage flux. The voltage and current ratios do not apply although the secondary current is directly proportional to the primary.

Current transformers must be short-circuited at all times since very high voltages can appear across their terminals.

