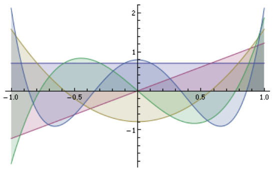
İZMİR UNIVERSITY OF ECONOMICS
Faculty of Engineering
EEE 281 Engineering Mathematics I
Fall 2023/2024



FINAL EXAM
Jan 5, 2024
120 min

Information on exam rules

Electronic devices such as laptops, mobile phones, and smartwatches are generally prohibited in the examination room. However, exceptions can be made for individuals with special needs, provided they have valid medical documentation. Requests for exceptions must be submitted with prior written approval from the academic advisor, and they should include details on the necessary measures to maintain the integrity and security of the examination.

Please refrain from engaging in cheating or any other prohibited activities during the examination. Suspected cheating may result in a score of zero on your exam, and any students found cheating may face disciplinary actions in accordance with law #2547. This includes actions such as using unauthorized electronic devices, communicating with classmates, exchanging exam or formula sheets, or using unauthorized written materials during the exam, all of which qualify as attempted cheating.

Declaration

I affirm that the activities and assessments completed as part of this examination are entirely my own work and comply with all relevant rules regarding copyright, plagiarism, and cheating. I acknowledge that if there is any question regarding the authenticity of any portion of my assessment, I may be subject to oral examination. The signatory of evidence records may also be contacted, or a disciplinary process may be initiated as per law #2547.

Signature of Student:

Last Name :.......................

Name :.......................

Group :.......................

Student No :.......................

Question	Points	Page	Grade
1	15		
2	15		
3	20		
4	20		
5	15		
6	15		
Total	100		

Q1. (*15 pts*) Consider the following ordinary differential equation. Determine the solution corresponding to the given initial conditions.

$$y'' + 4\,y' + 8\,y = 0, \qquad y(0) = 2, y'(0) = 4$$

Q2. (*15 pts*) Consider the following differential equation where *y* is a function of *x*.

$$4xyy' = 2y^2 - 1x^2$$

(i) Put the above differential equation into a separable form through introducing a new variable u as

$$u = \frac{4y}{x}$$

(ii) Solve this separable differential equation for the given initial condition $y(1) = 0$.

Q3. (*15 pts*) Consider the following ordinary differential equation (y is a function of t)

$$y'' + 3\,y' + 2\,y = 2\,e^{-t}$$

(i) Find the homogeneous solution

(ii) Determine the particular solution.

(iii) Determine the total solution corresponding to the initial conditions

$y(0) = 0, y'(0) = 0$

Q4. (*15 pts*) Consider the following differential equation.

$$5y\,dx + 2x\,dy = 0$$

(i) Check whether the above differential equation is exact or not. Show the details of your work.

(ii) Determine the value of $\boldsymbol{m}$ in order the following function is an integrating factor to put the above differential equation into exact form

$$F(x, y) = y\,x^m$$

(iii) Find the solution for the initial value of $y(1) = 2$

Q5. (*15 pts*) The paths $\boldsymbol{C_1}$ and $\boldsymbol{C_2}$ over which the following line integrals are to be calculated are as defined as:

- The path $\boldsymbol{C_1}$ is the semicircle centered at (0,0) with radius r = 2. The path $\boldsymbol{C_1}$ may be represented as $\boldsymbol{C_1}$: $z = 2\,e^{i\theta}, -\frac{\pi}{3} \le \theta \le \frac{\pi}{3}$.
- The closed contour $\boldsymbol{C_2}$ is the circle centered at (0,0) with radius r = 2. The closed contour $\boldsymbol{C_2}$ may be represented as $\boldsymbol{C_2}$: $z = 2\,e^{i\theta}, 0 \le \theta \le 2\pi$.

(i) Calculate the following integral over the path $\boldsymbol{C_1}$

$$\int_{C_1} \left(\frac{1}{z}\right) dz$$

(ii) Calculate the following integral over the closed contour $\boldsymbol{C_2}$

$$\oint_{C_2} \left(\frac{1}{z}\right) dz$$

(iii) Calculate the following integral over the path $\boldsymbol{C_1}$

$$\int_{C_1} z\, dz$$

(iv) Calculate the following integral over the closed contour $\boldsymbol{C_2}$

$$\oint_{C_2} z\, dz$$

(v) Comment on the results you obtained in part (ii) and (iv).

Q6. (*15 pts*) Determine $e^{\boldsymbol{A}t}$ if

$$\boldsymbol{A} = \begin{bmatrix} 2 & 0 \\ 2 & 1 \end{bmatrix}$$

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