



### MIDTERM EXAM 1

Nov 5, 2023

120 min

#### Information on exam rules

Electronic devices such as laptops, mobile phones, and smartwatches are generally prohibited in the examination room. However, exceptions can be made for individuals with special needs, provided they have valid medical documentation. Requests for exceptions must be submitted with prior written approval from the academic advisor, and they should include details on the necessary measures to maintain the integrity and security of the examination.

Please refrain from engaging in cheating or any other prohibited activities during the examination. Suspected cheating may result in a score of zero on your exam, and any students found cheating may face disciplinary actions in accordance with law #2547. This includes actions such as using unauthorized electronic devices, communicating with classmates, exchanging exam or formula sheets, or using unauthorized written materials during the exam, all of which qualify as attempted cheating.

#### Declaration

I affirm that the activities and assessments completed as part of this examination are entirely my own work and comply with all relevant rules regarding copyright, plagiarism, and cheating. I acknowledge that if there is any question regarding the authenticity of any portion of my assessment, I may be subject to oral examination. The signatory of evidence records may also be contacted, or a disciplinary process may be initiated as per law #2547.

**Signature of Student:**

Last Name :..... Name :..... Group :..... Student No :.....	Question	Points	Grade
	1	30	
	2	25	
	3	20	
	4	25	
	TOTAL	100	

Useful Trigonometric Values	
$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$	$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$
$\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$	$\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$
$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$	$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

**Q1. (30 pts)**

(i) (7 pts) Determine the real and the imaginary parts of  $z$  where

$$z = \frac{-2 - 3i}{4 + 3i}$$

(ii) (8 pts) Determine the real and the imaginary parts of  $z$  where

$$z = \sqrt{1 - i\sqrt{3}}$$

(iii) (7 pts) (a) Determine the roots of the following equation. (b) Verify your results.

$$-4z^2 + 4iz + 1 = 0$$

(iv) (8 pts) Determine the root(s) of the following equation.

$$\sqrt{z} + i\sqrt{z} = \sqrt{2} e^{i(3\pi/4)}$$

**Q2. (25 pts)**

(i) (10 pts) Determine  $a$  so that the function given below is harmonic.

$$v(x,y) = a x^3 + x y$$

(ii) (15 pts) Find its harmonic conjugate using the value of  $a$  found in part (i).

**Q3. (20 pts)** Calculate the following integral

$$\int_C \left(\frac{18}{\pi}\right) dz$$

over the paths defined below. (Note that  $z = x + iy$ )

(i) (10 pts) C: The shortest path from  $0 + 0i$  to  $3 + 0.5i$ .

(ii) (10 pts) C: The sine function  $y = \sin\left(\frac{\pi}{18}x\right)$  from  $0 + 0i$  to  $3 + 0.5i$ .

**Q4. (25 pts)** Assume  $z_1 = 3$  and  $z_2 = 3i$

- (i) (5 pts) Evaluate the following integral for any closed contour enclosing only the point  $z_1 = 3$ .

$$\int_{C_{\square}} \frac{z}{(z-z_1)(z-z_2)} dz \quad (z_1 = 3, z_2 = 3i)$$

- (ii) (5 pts) Evaluate the above integral for any closed contour enclosing only the point  $z_2 = 3i$ .  
(iii) (10 pts) Evaluate the above integral for any contour enclosing both the points  $z_1$  and  $z_2$ .  
(iv) (5 pts) Evaluate the above integral for any contour not containing the points  $z_1$  and  $z_2$ .