İZMİR UNIVERSITY OF ECONOMICS
Faculty of Engineering
EEE 281 Engineering Mathematics I
Fall 2023/2024

MIDTERM EXAM 1
Dec 3, 2023
120 min

Information on exam rules

Electronic devices such as laptops, mobile phones, and smartwatches are generally prohibited in the examination room. However, exceptions can be made for individuals with special needs, provided they have valid medical documentation. Requests for exceptions must be submitted with prior written approval from the academic advisor, and they should include details on the necessary measures to maintain the integrity and security of the examination.

Please refrain from engaging in cheating or any other prohibited activities during the examination. Suspected cheating may result in a score of zero on your exam, and any students found cheating may face disciplinary actions in accordance with law #2547. This includes actions such as using unauthorized electronic devices, communicating with classmates, exchanging exam or formula sheets, or using unauthorized written materials during the exam, all of which qualify as attempted cheating.

Declaration

I affirm that the activities and assessments completed as part of this examination are entirely my own work and comply with all relevant rules regarding copyright, plagiarism, and cheating. I acknowledge that if there is any question regarding the authenticity of any portion of my assessment, I may be subject to oral examination. The signatory of evidence records may also be contacted, or a disciplinary process may be initiated as per law #2547.

Signature of Student:

Last Name :……

Name :……

Group :……

Student No :……

Question	Points	Grade
1	30	
2	25	
3	20	
4	25	
TOTAL	100	

Useful Trigonometric Values	
$cos\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$	$sin\left(\frac{\pi}{6}\right)=\frac{1}{2}$
$cos\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}=\frac{1}{\sqrt{2}}$	$sin\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}=\frac{1}{\sqrt{2}}$
$cos\left(\frac{\pi}{3}\right)=\frac{1}{2}$	$sin\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}$

Q1. (20 pts) Let

$$\boldsymbol{A} = \begin{bmatrix} 3 & 1 & 4 \\ 2 & -3 & 3 \\ 4 & 0 & 2 \end{bmatrix},\ \boldsymbol{B} = \begin{bmatrix} 2 & 1 \\ -1 & 3 \\ 0 & -2 \end{bmatrix},\ \boldsymbol{C} = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix},\ \boldsymbol{D} = \begin{bmatrix} 4 \\ -2 \\ 3 \end{bmatrix},\ \boldsymbol{E} = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$$

Find the following expressions or give reasons why they are not defined.

(i) $\boldsymbol{A}\,\boldsymbol{B}$

(ii) $\boldsymbol{A}\,\boldsymbol{C}$

(iii) $\boldsymbol{B}^T\boldsymbol{A}$

(iv) $\boldsymbol{D}\,\boldsymbol{E}^T$

(v) $\det(\boldsymbol{C})$

(vi) $\boldsymbol{C}^{-1}$

Q2. Consider the linear system of equations given below.

(a) $a_{11}\,x_1 + a_{12}\,x_2 + a_{13}\,x_3 = b_1$ $a_{21}\,x_1 + a_{22}\,x_2 + a_{23}\,x_3 = b_2$ $a_{31}\,x_1 + a_{32}\,x_2 + a_{33}\,x_3 = b_3$	(b) $a_{11}\,x_1 + a_{12}\,x_2 = c_1$ $a_{21}\,x_1 + a_{22}\,x_2 = c_2$

(i) For each set of equations, obtain the corresponding augmented matrix and solve the unknowns by Gauss Elimination method.

(ii) Which system has (a) exactly a single solution, (b) multiple solutions or (c) no solution?

(iii) Solve part (b) using Cramer's Rule and compare your results you obtained in part (i).

Q3. Consider the given matrix $\boldsymbol{A}$ and one of its eigenvectors $\boldsymbol{x}_1$.

$$\boldsymbol{A} = \begin{bmatrix} -\lambda_3 & 0 & -\lambda_3 \\ -2a\lambda_3 & \lambda_2 & -2a\lambda_3 \\ 2\lambda_3 & 0 & 2\lambda_3 \end{bmatrix}, \qquad \boldsymbol{x_1} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

(i) Show that $\boldsymbol{x}_1$ is an eigenvector and determine the corresponding eigenvalue.

(ii) Determine the other eigenvalues and the corresponding eigenvectors.

Q4. Consider the given matrix $\boldsymbol{P}$.

$$\boldsymbol{P} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & a \\ 1 & 0 & 1 \end{bmatrix}$$

(i) Determine $\boldsymbol{P^{-1}}$ and verify you result.

(ii) Calculate the matrix $\boldsymbol{P^{-1}}$ which defined as $\widehat{\boldsymbol{A}} = \boldsymbol{P^{-1}AP}$ Show that $\boldsymbol{x}_1$ is an eigenvector and determine the corresponding eigenvalue.