Take Home Exam 1
Complex Numbers

Q1. Determine $z = \frac{3+i4}{4-i3}$.

Q2. Determine $z = \sqrt{2 + i2\sqrt{3}}$

Q3. Show that

(i) $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$
(ii) $\overline{z_1 z_2} = \overline{z_1}\,\overline{z_2}$
(iii) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$

where $\bar{z}$ denotes the complex conjugate of the complex number z.

Q4. Determine $z = i^n$, n=0, 1, 2, 3,….

Q5. Find the value of z= $i^2 + i^4 + i^6 + \cdots + i^{2n}$?

Q6. Determine $z = \frac{\cos\theta + i\sin\theta}{\cos\theta - i\sin\theta}$.

Q7. Find the value of $z = (i - i^2)^3$?

Q8. If $z^3 = 1$, then determine $\mathrm{w} = z^6 + z^7 + z^5$.

Q9. If (1+i)(x+iy)=2+i4 then determine x.

Q10. Determine z if $w = \frac{z-1}{z+1}$ is purely imaginary.

(Bonus). Using Taylor series expansion show that $e^{i\theta} = \cos\theta + i\sin\theta$ (Euler's formula).