Take Home Exam 2

Complex Functions

Q1. Determine and sketch or graph the sets in the complex plane given by

(i) $|z-3-4i| \leq 5$

(ii) $\mathrm{Re}(1/z) \leq 1$

(iii) $|z+1| \geq |z-1|$

Q2. Find the value of the derivative of the following functions at the specified points.

(i) $f(z) = (3z^2 + iz)^3$ at $z = 0$.

(ii) $f(z) = \frac{z-1}{z+\mathrm{i}}$ at $z = 1$.

Q3. Are the following functions analytic? (Using proper form of Cauchy-Riemann Equations)

(i) $f(z) = \frac{z}{\bar{z}}$

(ii) $f(z) = Re(z^2) + i\, Im(z^2)$

(iii) $f(z) = \ln r + i\theta$ where $z = r\, e^{i\theta}$

Q4. Are the following functions harmonic? If your answer is yes, find a corresponding analytic function $f(z) = u(x,y) + i\, v(x,y)$

(i) $u(x,y) = x^3 + 3xy^2$

(ii) $v(x,y) = e^x cosy$

Q5. Determine a and b so that the given function is harmonic and find a harmonic conjugate.

(i) $u(x,y) = ax^3 + bxy$

(ii) $v(x,y) = \cosh ax \cos by$

Q-bonus. Determine z such that $\sin z = 5$.