EEE 281 Engineering Mathematics I
Take-Home Exam Part 3
Due: 31 Oct 2023

Take Home Exam 3
Line Integral of Complex Functions

Q1. Find and sketch the following paths.

(i) $(3+i)t \quad (1 \le t \le 2)$

(ii) $2+i+(1-i)t \quad (0 \le t \le 2)$

(iii) $1+i+4e^{it} \quad (0 \le t \le 2\pi)$

Q2. Find a parametric representation of the following paths and sketch them.

(i) Line segment from (0,-1) to (2,1).

(ii) Unit circle, clockwise

(iii) Ellipse $x^2 + 4\,y^2 = 16$, counterclockwise

Q3. Calculate the following integral

$$\int_C Re\, z\, dz$$

over the paths defined below.

(i) C: The shortest path from $1+i$ to $1+4i$.

(ii) C: The parabola $y = x^2$ from $1+i$ to $1+4i$.

Q4. Express $f(z)$ in terms of partial fractions and integrate it over the circle $|z| = 2$ (counterclockwise) where

(i) $f(z) = \frac{2z+1+i}{z^2+1}$

(ii) $f(z) = \frac{z+i}{z^2+4z}$

(iii) $f(z) = \frac{4z+1}{z^2+9}$

Q5. Calculate the following integral

$$\oint_C \left(\frac{2z+4-i}{(z-i)^2}\right) dz$$

over the closed path C defined as C: The circle $|z-i| = 1$, clockwise

Bonus. Integrate

$$\oint_C \frac{e^z}{z\,(z-1-i)}\, dz$$

where the contour C consists of the circle $|z| = 2$ (C_1) counterclockwise and the circle $|z| = 1$ (C_2) clockwise. (C_1 and C_1 constitute the boundary for the ring-shaped domain)