Take Home Exam 4

Linear System of Equations and Matrices

Q1. Let

$$\boldsymbol{A}=\begin{bmatrix}3 & 1 & 4\\ 2 & -3 & 3\\ 4 & 0 & 2\end{bmatrix},\ \boldsymbol{B}=\begin{bmatrix}2 & 3 & -1\\ 0 & 4 & 3\\ 3 & -1 & 1\end{bmatrix},\ \boldsymbol{C}=\begin{bmatrix}2 & 1\\ -1 & 3\\ 0 & -2\end{bmatrix},\ \boldsymbol{D}=\begin{bmatrix}4\\ -2\\ 3\end{bmatrix},\ \boldsymbol{E}=\begin{bmatrix}3\\ 2\\ -1\end{bmatrix}$$

Find the following expressions or give reasons why they are not defined.

(i) $\boldsymbol{A}+\boldsymbol{B}$
(ii) $\boldsymbol{A}+\boldsymbol{C}$
(iii) $\boldsymbol{A}+\boldsymbol{D}$
(iv) $\boldsymbol{D}+\boldsymbol{E}$
(v) $\boldsymbol{A}^T$
(vi) $\boldsymbol{D}^T$

Q2. Let the matrices $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{D}$ and $\boldsymbol{E}$ are as given in ***Q1***. Find the following expressions or give reasons why they are not defined.

(i) $\boldsymbol{A}\,\boldsymbol{B}$
(ii) $\boldsymbol{A}\,\boldsymbol{C}$
(iii) $\boldsymbol{C}\,\boldsymbol{A}$
(iv) $\boldsymbol{D}\,\boldsymbol{A}$
(v) $\boldsymbol{A}\,\boldsymbol{E}$
(vi) $\boldsymbol{D}^T\boldsymbol{E}$

Q3. (a) **Idempotent matrix**, defined by $\boldsymbol{A}^2=\boldsymbol{A}$**.** Can you find two 2x2 idempotents matrices? Verify your suggestion. What is the property of an idempotents matrix?

(b) **Nilpotent matrix**, defined by $\boldsymbol{A}^m=\boldsymbol{0}$ for some $\boldsymbol{A}$. Can you find two 2x2 nilpotent matrices? Verify your suggestion. What is the property of a nilpotents matrix??

(c) Let $\boldsymbol{A}$ be the matrix as given in **Q1**.

(i) Show that $\boldsymbol{S}=\boldsymbol{A}+\boldsymbol{A}^{\boldsymbol{T}}$ is symmetric.

(ii) Show that $\boldsymbol{T}=\boldsymbol{A}-\boldsymbol{A}^{\boldsymbol{T}}$ is skew-symmetric.

(iii) Show that $\boldsymbol{A}$ can be expressed as the sum of symmetric and skew-symmetric components $\boldsymbol{A}=\frac{1}{2}(\boldsymbol{S}+\boldsymbol{T})$

Q4. Consider the linear system of equations given below. For each part, obtain the corresponding augmented matrix and solve the unknowns by Gauss Elimination method. Which system has (a) exactly a single solution, (b) multiple solutions or (c) no solution?

(i) $4\,x_1-6\,x_2=-14$
$3\,x_1+8\,x_2=2$

(ii) $4\,x_1+6\,x_2=12$
$2\,x_1+3\,x_2=5$

(iii) $4\,x_1+6\,x_2=10$
$2\,x_1+3\,x_2=5$

(iv) $3\,x_1+x_2+x_3=9$
$x_1-3\,x_2-2\,x_3=5$
$-x_1+x_2-2\,x_3=-9$