

Q1: a) $\begin{bmatrix} 4 & -6 \\ 3 & -8 \end{bmatrix} \rightarrow D = -32 + 18 = -14$

$D_1 = \begin{vmatrix} -14 & -6 \\ 2 & -8 \end{vmatrix} = 124$ $D_2 = \begin{vmatrix} 4 & -14 \\ 3 & 2 \end{vmatrix} = 8 + 42 = 50$

$x_1 = -\frac{62}{7}$

$x_2 = -\frac{50}{7}$

b) $\begin{bmatrix} 3 & 1 & 1 \\ 1 & -3 & -2 \\ -1 & 1 & -2 \end{bmatrix} \rightarrow D = 22$

$D_1 = \begin{vmatrix} 9 & 1 & 1 \\ 5 & -3 & -2 \\ -9 & 1 & -2 \end{vmatrix} = 78$ $D_2 = \begin{vmatrix} 3 & 9 & 1 \\ 1 & 5 & -2 \\ -1 & -9 & -2 \end{vmatrix} = -52$ $D_3 = \begin{vmatrix} 3 & 1 & 9 \\ 1 & -3 & 5 \\ -1 & 1 & -9 \end{vmatrix} = 43$

$x_1 = \frac{78}{22} = \frac{39}{11}$

$x_2 = -\frac{52}{22}$

$x_3 = \frac{43}{22}$

Q2: $A = \begin{bmatrix} 2 & -2 \\ 1 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 3 & 1 \\ 3 & 4 \end{bmatrix}$

a) $C = AB = ?$

$C = \begin{bmatrix} 0 & -6 \\ 15 & 17 \end{bmatrix}$

b) $A^{-1} = ?$

$A^T = \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix}$

$\det A = 10$

$A^{-1} = \frac{1}{10} \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix}$

$B^{-1} = ?$

$B^T = \begin{bmatrix} 3 & 3 \\ 1 & 4 \end{bmatrix}$

$\det B = 9$

$\frac{1}{9} \begin{bmatrix} 3 & 3 \\ 1 & 4 \end{bmatrix} = B^{-1}$

$C^{-1} = ?$

$C^T = \begin{bmatrix} 0 & 15 \\ -6 & 17 \end{bmatrix}$

$\det = 90$

$C^{-1} = \frac{1}{90} \begin{bmatrix} 0 & 15 \\ -6 & 17 \end{bmatrix}$

c) $\frac{1}{90} \begin{bmatrix} 3 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 15 \\ -6 & 17 \end{bmatrix} \frac{1}{90} \quad (\checkmark)$

d) $\frac{10}{\det A} \times \frac{9}{\det B} = \frac{90}{\det C}$

Q3: (i) $\rightarrow A = \begin{bmatrix} 2 & -2 \\ 1 & 4 \end{bmatrix}$

$$\left[\begin{array}{cc|cc} 2 & -2 & 1 & 0 \\ 1 & 4 & 0 & 1 \end{array} \right] \rightarrow \left[\begin{array}{cc|cc} 1 & -1 & 0.5 & 0 \\ 1 & 4 & 0 & 1 \end{array} \right] \rightarrow \left[\begin{array}{cc|cc} 1 & -1 & 0.5 & 0 \\ 0 & 5 & -0.5 & 1 \end{array} \right]$$

$$\left[\begin{array}{cc|cc} 1 & -1 & 0.5 & 0 \\ 0 & 1 & -0.1 & 0.2 \end{array} \right] \rightarrow \left[\begin{array}{cc|cc} 1 & 0 & 2/5 & 1/5 \\ 0 & 1 & -0.1 & 1/5 \end{array} \right] \quad A^{-1} = \begin{bmatrix} 2/5 & 1/5 \\ -1/10 & 1/5 \end{bmatrix}$$

(ii) $\rightarrow \left[\begin{array}{ccc|ccc} 3 & 1 & 1 & 1 & 0 & 0 \\ 1 & -3 & -2 & 0 & 1 & 0 \\ -1 & 1 & -2 & 0 & 0 & 1 \end{array} \right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & -3 & -2 & 0 & 1 & 0 \\ 0 & 1 & 7 & 1 & -3 & 0 \\ 0 & -2 & -4 & 0 & 1 & 1 \end{array} \right]$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 4/13 & 3/26 & 1/26 \\ 0 & 1 & 0 & 2/13 & -5/26 & 7/26 \\ 0 & 0 & 1 & -1/13 & -2/13 & -5/13 \end{array} \right]$$

Q4: $a = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ $b = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ $c = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $d = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$ $e = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

for (a, b) , (a, c) , (b, d)

$(a, b) \rightarrow$ independent

$$\begin{bmatrix} 1 \\ -3 \end{bmatrix} \begin{bmatrix} 3 & 1 \end{bmatrix} = 0$$

$\hookrightarrow$ orthogonal

$$\|d\| = \sqrt{6^2 + 2^2} = 2\sqrt{10}$$

$$d = \frac{1}{2\sqrt{10}} \begin{bmatrix} 6 \\ 2 \end{bmatrix}$$

$(a, c) \rightarrow$ independent

$$\begin{bmatrix} 1 \\ -3 \end{bmatrix} \begin{bmatrix} 1 & 2 \end{bmatrix} = \begin{bmatrix} -5 \end{bmatrix}$$

$\hookrightarrow$ Not orthogonal

$(b, d) \rightarrow$ dependent

$$\begin{bmatrix} 3 \\ 1 \end{bmatrix} \begin{bmatrix} 6 & 2 \end{bmatrix} = 20$$

$\hookrightarrow$ Not orthogonal

$$d = \frac{1}{2\sqrt{10}} \begin{bmatrix} 6 \\ 2 \end{bmatrix}$$