Q1

HOMEWORK 7 - ANSWER KEY - EEE 281

a) $y' - 4\cos(2\pi x) = 0$

$$y' = 4\cos(2\pi x)$$
$$\int y' dy = \int 4\cos(2\pi x)\,dx$$

$$y = \frac{4.\sin(2\pi x)}{2\pi} + c$$
$$y = \frac{2\sin(2\pi x)}{\pi} + c$$

b) $y' + x.e^{-x^2} = 0$

$$y' = -x.e^{-x^2}$$
$$\int y' dy = \int(-x.e^{-x^2})\,dx$$

$$x^2 = t$$
$$2x.dx = dt$$
$$\int y' dy = \int -\frac{1}{2}e^{-t}dt$$

$$y = -\frac{1}{2}\left[\int e^{-t}dt\right]$$
$$y = -\frac{1}{2}(-e^{-t})$$

$$y = \frac{1}{2}e^{-t} + C$$

c) $y' = -2y$

$$\frac{y'}{y} = -2 = \frac{dy}{y}$$

$$\int \frac{1}{y}dy = \int -2dx$$
$$\ln y = -2x + C$$

$$y = Ae^{-2x}$$

Q2 : (verification)

a) $y' + 2xy = 4$, $y = c.e^{-x^2}$, $y(0) = 2$

$$y' = -2cx.e^{-x^2}$$
$$\underbrace{-2cx.e^{-x^2} + 2x.c.e^{-x^2}}_{0} = 4$$
$$0 \neq 4$$

Equation is not True. Thats why, there is not a solution to the ODE.

b) $y' = 2(y - y^2)$; $y = \frac{1}{1 + c.e^{-2x}}$; $y(0) = 0.5$

$$y' = -\frac{-2c.e^{2x}}{(1 + c.e^{-2x})^2}$$

$$y' = 2(y - y^2)$$

$$\frac{2ce^{-2x}}{(1+ce^{-2x})^2} = 2\left(\frac{1}{1+ce^{-2x}} - \frac{1}{(1+c.e^{-2x})^2}\right)$$

$$\frac{\cancel{2}.c.e^{-2x}}{(1+c.e^{-2x})^2} = \cancel{2}\left[\frac{1+c.e^{-2x}-1}{(1+c.e^{-2x})^2}\right]$$ $\Rightarrow$ This equation is true. There is a solution to the ODE.

If $y(0) = 0.5$

$$y = \frac{1}{1+c.e^{-2x}} \Rightarrow 0.5 = \frac{1}{1+c} \Rightarrow 0.5 + 0.5c = 1 \quad c = 1$$

$$0.5c = 0.5$$

Particular Solution $\Rightarrow$ $y = \dfrac{1}{1+e^{-2x}}$

c) $y' = 2y + e^{2x}$, $y = (x+c)e^{2x}$, $y(0) = 1$

$$y' = e^{2x} + 2(x+c)e^{2x} = e^{2x}(1+2x+2c)$$

$$e^{2x}(1+2x+2c) = 2(x+c)e^{2x} + e^{2x}$$

$$e^{2x}(1+2x+2c) = e^{2x}(2x+2c+1)$$ $\Rightarrow$ This equation is true.

There is a solution to the ODE

$$\left.\begin{array}{l} y = (x+c)e^{2x} \\ x = 0 \rightarrow y = 1 \end{array}\right\} 1 = c$$

particular solution $\Rightarrow$ $y = (x+1)e^{2x}$

Q3: $(y')^2 - x.y' + y = 0$ has a singular solution.

general solution $\Rightarrow$ $y = c.x - c^2$ singular solution $\Rightarrow$ $y = x^2/4$

$(y')^2 - x.y' + y = 0$

$$\left.\begin{array}{l} y = c.x - c^2 \\ y' = c \end{array}\right\} \cancel{c^2} - \cancel{c}.x + \cancel{c}x - \cancel{c^2} = 0 \checkmark$$

so; $y = c.x - c^2$ is a solution of the differential equation $(y')^2 - x.y' + y = 0$

$y = x^2/4 \rightarrow y' = \frac{1}{2}.x$

$$\frac{x^2}{4} - x^2.\frac{1}{2} + \frac{x^2}{4} = 0 \checkmark$$

$y = x^2/4$ is a singular solution of differential equation.

Q4: $y' = k.y$; y = amount of radioactive element ; ${}^{224}_{88}Ra$ has a half-life of about 4 days.

t = time k = decay constant.

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a) given 1 gram, how much will still be present after 1 day and 1 year. (b)

In the context of r.a. decay, the relationship between k and half-life (Thalf) is $k = \frac{\ln(2)}{T_{half}}$ $\Rightarrow$ half life of 4 days

$$k = \frac{\ln(2)}{4}$$

a) After 1 day

$$\frac{dy}{dt} = -\frac{\ln(2)}{4} \cdot y \Rightarrow \frac{dy}{y} = -\frac{\ln(2)}{4} \cdot dt$$

$$\underbrace{\int \frac{1}{y} \cdot dy}_{\ln(y)} = \underbrace{-\int k \cdot dt}_{-k.t} \Rightarrow y = e^{-k.t} = e^{-\left(\frac{\ln 2}{4}\right)t}$$

for 1 day: $y_1 = e^{-\frac{\ln 2}{4}}$

for 1 year: $y_{365} = e^{-\frac{\ln 2}{4} \cdot 365}$