Take Home Exam 7
Ordinary Differential Equations (ODEs)

Q1. (*Solving Simple Differential Equations*) Solve the following ordinary differential equations (ODEs) by simple integration.

(i) $y' - 4\cos 2\pi x = 0$

(ii) $y' + x\, e^{-x^2} = 0$

Q2. (*Verification of a solution*) Verify that y is a solution of the given ODE and determine from y the particular solution corresponding to the initial value given.

(i) $y' + 2\,x\,y = 4,\ y = c\ e^{-x^2},\ y(0) = 2$

(ii) $y' = 2\,(y - y^2),\ y = \frac{1}{1 + c\, e^{-2x}}\ ,\ y(0) = 0.5$

Q3. (*Half-life*) Radioactive decay is governed by the ODE $y' = k\,y$ where y is the amount of radioactive element. Radium ${}^{224}_{88}Ra$ has a half-life of about 4 days.

(a) Given 1 gram, how much will still be present after 1 day?
(b) After 1 year?

Q4. (*Separable ODEs*) Find a general solution. Check your answer by substitution.

(i) $y^3\,y' - x^2 = 0$

(ii) $y' = (\sec y)^2$

Q5. (*Reduction to Separable Form*) By setting $u = y/x$, find a general solution. If an initial value is given, find the corresponding particular solution.

(i) $x\,y' = y^2 + y$

(ii) $x\,y' = y + 3\,x^4\cos^2(y/x)\,,\ y(1) = 0$

Q6. (*Reduction to Separable Form*) By setting u as suggested, find a general solution. If an initial value is given, find the corresponding particular solution.

(i) $y' = (y + 4\,x)^2$ (Set $u = y + 4\,x$)

(ii) $y' = (x + y - 2)^2,\ y(0) = 2$ (Set $u = x + y - 2$)