



Take Home Exam 8
Separable and Exact Differential Equations

- Q1.** (*Separable ODEs*) Find a general solution. Check your answer by substitution.
- (i) $y^3 y' - x^2 = 0$
 - (ii) $y' = (\sec y)^2$
 - (iii) $y' = 4 e^{4x-1} y^2$
- Q2.** (*Reduction to Separable Form*) By setting $u = y/x$, find a general solution. If an initial value is given, find the corresponding particular solution.
- (i) $x y' = y^2 + y$
 - (ii) $x y' = x + y$
 - (iii) $x y' = y + 3 x^4 \cos^2(y/x), y(1) = 0$
- Q3.** (*Reduction to Separable Form*) By setting u as suggested, find a general solution. If an initial value is given, find the corresponding particular solution.
- (i) $y' = (y + 4 x)^2$ (Set $u = y + 4 x$)
 - (ii) $y' = (x + y - 2)^2, y(0) = 2$ (Set $u = x + y - 2$)
- Q4.** (*Exact Differential Equations*). Test for exactness and if exact solve.
- (i) $2 xy dx + x^2 dy = 0$
 - (ii) $x^3 dx + y^3 dy = 0$
 - (iii) $\sin x \cos y dx + \cos x \sin y dy = 0$
 - (iv) $e^{3\theta} (dr + 3r d\theta) = 0$
- Q5.** (*Reduction to Exact Form-Integrating Factors*) Test for the exactness, if exact solve. If not use the integrating factor as given and check for the exactness and solve.
- (i) $3 (y + 1) dx = 2x dy; F = (y + 1) x^{-4}$
 - (ii) $y dx + [y + \tan(x + y)] dy = 0; F = \cos(x + y)$
- Q6.** (*Exact Form and Particular Solution*) Test for the exactness, if exact solve. If not find the integrating factor and check for the exactness. Find the particular solution corresponding to the initial condition given.
- (i) $(2xydx + dy) e^{x^2} = 0; y(0) = 2$
 - (ii) $(a + 1) y dx + (b + 1) x dy = 0; y(0) = 1; F = x^a y^b;$
- Q7.** (*Exactness*) Under what conditions for the constants a, b, k , and l is the differential equation

$$(ax + by) dx + (kx + ly) dy = 0$$

is exact? Solve the exact ODE.