HOMEWORK 9 - SOLUTION - EEE281

Q1: $y'' + 6y' + 8y = 12\sin(2t)$

$\Rightarrow$ Homogenous: $y'' + 6y' + 8y = 0$

$r^2 + 6r + 8 = 0$

$r_1 = -4, \quad r_2 = -2$

$y_h(t) = c_1 e^{-4t} + c_2 e^{-2t}$

$\Rightarrow$ Particular: $y_p(t) = A\sin(2t) + B\cos(t)$

$y_p' = 2A\cos(2t) + 2B\sin(2t)$

$y_p'' = -4A\sin(2t) - 4B\cos(2t)$

$(4A - 12B)\sin 2t + (12A + 4B)\cos 2t = 12\sin 2t$

$12A + 4B = 0 \qquad 4A - 12B = 12$

$3A = -B \qquad 4A + 36A = 12 \qquad A = 3/10$

$B = -9/10$

$y_p = \frac{3}{10}\sin(2t) - \frac{9}{10}\cos(2t)$

$\Rightarrow$ Total $= y_h(t) + y_p(t)$

Q2: $y'' + 5y' + 4y = 10e^{-3x}$

Homogenous: $r^2 + 5r + 4 = 0$

$r_1 = -4 \quad r_2 = -1$

$y_h = c_1 e^{-4t} + c_2 e^{-t}$

Particular: $y_p = Ae^{-3x}$

$y_p' = -3Ae^{-3x}$

$y_p'' = 9Ae^{-3x}$

$9Ae^{-3x} - 15e^{-3x}A + 4Ae^{-3x} = 10e^{-3x} \qquad A = -5$

$y_p = -5e^{-3x}$

Total $= y_h + y_p$

Q3: $y'' + 3y' + 2y = 2x^2$

Homogenous: $r^2 + 3r + 2 = 0$

$r = -1 \quad r = -2$

$y_h = c_1 e^{-t} + c_2 e^{-2t}$

Particular $\Rightarrow y_p = K_2 x^2 + K_1 x + K_0$

$y_p' = 2K_2 x + K_1$

$y_p'' = 2K_2$

$2K_2 + 6K_2 x + 3K_1 + 2K_2 x^2 + 2K_1 x + 2K_0 = 2x^2$

$K_2 = 1 \qquad K_1 = -3 \qquad K_0 = 7/2$

$y_p = x^2 - 3x + 7/2$

$y_T = y_h + y_p$

Q4: $y'' + 4y' + 4y = e^{-x} \cdot \cos x$

Homogeous: $\lambda^2 + 4\lambda + 4 = 0$

$\lambda_1 = -2 \quad \lambda_2 = -2$

$y_h = (C_1 + C_2) e^{-2x} = C . e^{-2x}$

$y_h + y_p = y_T$

Particular: $y_p = e^{-x}(K.\cos x + M.\sin x)$

$y_p' = -e^{-x}(K.\cos x + M.\sin x)$
$+ e^{-x}(-K\sin x + M.\cos x)$

$y_p'' = e^{-x}(-2M.\cos x + 2K.\sin x)$

$K = 0 \qquad M = 1/2$

$y_p = e^{-x}(\frac{1}{2}\sin x)$

Q5; $y'' + 4y' + 8y = e^{-x}$

Homogeous

$\lambda^2 + 4\lambda + 8 = 0$

$\lambda_{1,2} = -2 \pm 2i$

$y_h = e^{2x}(C_1.\cos 2x + C_2 \sin 2x)$

$y_T = y_p + y_h$

Particular: $y_p = K.e^{-x}$

$y_p' = -K.e^{-x}$

$y_p'' = K.e^{-x}$

$K.e^{-x} - 4K.e^{-x} + 8Ke^{-x} = -e^{-x}$

$5K = 1 \quad K = -1/5$

$y_p = \frac{1}{5} e^{-x}$

Q6: $y'' + 3y' + 2y = 0 \qquad y(0) = 2 \quad y'(0) = 1$

$\lambda^2 + 3\lambda + 2 = 0$

$\lambda = -1 \quad \lambda = -2$

$y_h = C_1 . e^{-2x} + C_2 e^{-x}$

$y'_h = -2C_1 . e^{-2x} - C_2 . e^{-x}$

$y(0) = C_1 + C_2 = 2$

$y'(0) = -2C_1 - C_2 = 1$

$-C_1 = 3 \quad C_1 = -3$

$C_2 = 5$

$y = -3e^{-2x} + 5e^{-x}$

Q7: $y'' + 2y' + 2y = 0 \quad y(0)=1 \quad y'(0)=0$

$\lambda^2 + 2\lambda + 2 = 0$

$\lambda_{1,2} = -1 \pm i$

$y = e^{-x}.(c_1.\cos x + c_2.\sin x) \rightarrow y' = -e^{-x}(c_1.\cos x + c_2 \sin x)$

$y(0) = c_1 = 1 \qquad + e^{-x}(-c_1.\sin x + c_2.\cos x)$

$y'(0) = -c_1 + c_2 = 0 \quad c_2 = 1$

$y = e^{-x}(\cos x - \sin x)$

Q8: $y'' + 4y = 8x^2 \quad y(0) = -2 \quad y'(0) = 0$

Homogeneous

$\lambda^2 + 4 = 0$

$\lambda_1 = 2i \quad \lambda_2 = -2i$

$y_h = c_1.\cos 2x + c_2 \sin 2x$

particular

$y_p = K_2.x^2 + K_1.x + K_0$

$y_p' = 2K_2 x + K_1$

$y_p'' = 2K_2$

$2K_2 + 4K_2.x^2 + 4K_1.x + 4K_0 = 8x^2$

$K_2 = 2 \quad K_1 = 0 \quad 4K_0 + 4 = 0 \quad K_0 = -1$

$y_p = 2x^2 - 1$

$y_h = c_1.\cos 2x + c_2.\sin 2x$

$y_T = c_1.\cos 2x + c_2 \sin 2x + 2x^2 - 1 \rightarrow y(0) = c_1 - 1 = -2 \quad c_1 = -1$

$y'_T = -2c_1.\sin 2x + 2c_2.\cos 2x + 4x \rightarrow y'(0) = 2c_2 = 0 \quad c_2 = 0$

$y_T = -\cos 2x + 2x^2 - 1$

Q9: $y'' + 9y = 2\sin 3x \qquad y(0) = 2 \qquad y'(0) = 4$

Homogeneous

$\lambda^2 + 9 = 0$

$\lambda_{1,2} = \mp 3i$

$y_h = C_1 \cos 3x + C_2 \sin 3x$

Particular

$$y_p = -\frac{x\cos(3x)}{3} + \frac{\sin(3x)}{9}$$

$$y_T = y_h + y_p = C_1\cos(3x) + C_2\sin(3x) - \frac{x\cos(3x)}{3} + \frac{\sin(3x)}{9}$$

$$y_T' = -3C_1\sin(3x) + 3C_2\cos(3x) - \left[\frac{\cos(3x) - 3x\sin(3x)}{3}\right] + \frac{1}{3}\cos(3x)$$

$y(0) = 2 \Rightarrow C_1 = 2$

$y'(0) = 4 \Rightarrow 3C_2 - \frac{1}{3} + \frac{1}{3} = 4 \rightarrow C_2 = 4/3$

$$y_T = 2\cos(3x) + \frac{4}{3}\sin(3x) - \frac{x}{3}\cos(3x) + \frac{\sin(3x)}{9}$$

Q10: $y'' + 4y' + 4y = e^{-2x}\sin(2x) \qquad y(0) = 1 \qquad y'(0) = -1$

Homogeneous

$\lambda^2 + 4\lambda + 4 = 0$

$\lambda_{1,2} = -2$

$y_h = C_1 e^{-2x} + C_2 e^{-2x}$

Particular

$y_p = A e^{-2x}\sin(2x) + B e^{-2x}\cos(2x)$

$y_p' = A\left[-2e^{-2x}\sin(2x) + e^{-2x}2\cos(2x)\right]$

$\quad + B\left[-2e^{-2x}\cos(2x) - 2e^{-2x}\sin(2x)\right]$

$$y_p' = e^{-2x}\sin(2x)\left[-2A-2B\right] + e^{-2x}\cos(2x)\left[2A-2B\right]$$

$$y_p'' = \left[-2A-2B\right]\left[-2e^{-2x}\sin(2x) + 2e^{-2x}\cos(2x)\right] + \left[2A-2B\right]\left[-2e^{-2x}\cos(2x) - 2e^{-2x}\sin(2x)\right]$$

$$\left.\begin{aligned} K &= e^{-2x}\sin 2x \\ M &= e^{-2x}\cos 2x \end{aligned}\right\} \quad \begin{aligned} y_p &= A K + B M \\ y_p' &= -2AK + 2MA - 2BM - 2BK \\ y_p'' &= [-2A-2B](-2K) + [-2A-2B](2M) + [2A-2B](-2)M \\ &\quad - [2A-2B](2)K \end{aligned}$$

$y_p = e^{-2x}\left(-\cos 2x - \frac{1}{4}\sin 2x\right) \qquad C_1 = 2 \qquad C_2 = -3/2$

$y_T = e^{-2x}\left[2 - 3/2\, x - \cos 2x - 1/4 \sin 2x\right]$