Linear ODEs of Second Order

$$y'' + p(x)\,y' + q(x)\,y = r(x)$$

Homogeneous: $r(x) = 0$

Nonhomogeneous: $r(x) \neq 0$

Homogeneous Linear ODEs of Second Order

$$y'' + p(x)\,y' + q(x)\,y = 0$$

Two solutions y_1 & y_2 exist. By superposition

$y = c_1 y_1 + c_2 y_2$ is also a solution

$$c_1 y_1'' + c_2 y_2'' + p(x)(c_1 y_1' + c_2 y_2') + q(x)(c_1 y_1 + c_2 y_2) = 0$$

$$c_1 \underbrace{\left[y_1'' + p(x) y_1' + q(x) y_1 \right]}_{0} + c_2 \underbrace{\left[y_2'' + p(x) y_2' + q(x) y_2 \right]}_{\neq 0} = 0$$

So $c_1 y_1 + c_2 y_2$ is also a solution.

Emphasis will be given to constant coefficient linear second order ODEs.

→ (146A)

Homogeneous second order linear ODEs with constant coefficients.

$$y'' + a y' + b y = 0$$

Assume

$$y = e^{\lambda x}$$

is a solution.

$$y' = \lambda e^{\lambda x}, \quad y'' = \lambda^2 e^{\lambda x}$$

Then

$$\lambda^2 e^{\lambda x} + a\lambda e^{\lambda x} + b e^{\lambda x} = 0$$

Linear Second Order ODE

$y'' + p(x)y' + q(x)y = r(x)$

$r(x) = 0 \Rightarrow$ Homogeneous ODE.

Superposition applies.

y_1 & y_2 $\Rightarrow$ $y = c_1 y_1 + c_2 y_2$ is also a solution

y_1 & y_2 must be independent. So are y_1' & y_2'

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \underbrace{y_1 y_2' - y_2 y_1'}_{\text{Wronskian}} \neq 0 \quad \text{Test for in.}$$

Example: $y'' + ay' + by = 0$

Assume $y = e^{\lambda x}$ is a solution

$y' = \lambda e^{\lambda x}$

$y'' = \lambda^2 e^{\lambda x}$

$$y'' + ay' + by = 0 \Rightarrow \lambda^2 e^{\lambda x} + a\lambda e^{\lambda x} + b e^{\lambda x} = 0$$

$$\underset{\overset{\|}{0}}{(\lambda^2 + a\lambda + b)} e^{\lambda x} \;\; 0$$

$$\lambda^2 + a\lambda + b = 0, \quad \lambda_{1,2} = \frac{-a \mp \sqrt{a^2 - 4b}}{2}$$

$$\lambda_1 = \frac{1}{2}(-a + \sqrt{a^2 - 4b}), \quad \lambda_2 = \frac{1}{2}(-a - \sqrt{a^2 - 4b})$$

$$\left.\begin{aligned} y_1 &= e^{\lambda_1 x}, \quad y_1' = \lambda_1 e^{\lambda_2 x} \\ y_2 &= e^{\lambda_2 x}, \quad y_2' = \lambda_2 e^{\lambda_2 x} \end{aligned}\right\} \quad W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^{\lambda_1 x} & e^{\lambda_2 x} \\ \lambda_1 e^{\lambda_1 x} & \lambda_2 e^{\lambda_2 x} \end{vmatrix}$$

$$W = \lambda_2 e^{\lambda_1 x} e^{\lambda_2 x} - \lambda_1 e^{\lambda_1 x} e^{\lambda_2 x} = (\lambda_2 - \lambda_1) e^{(\lambda_1 + \lambda_2)x} \neq 0 \quad \text{if } \lambda_1 \neq \lambda_2$$

If $\lambda_2 = \lambda_1$; we should suggest a different y_2 such as

$y_1 = e^{\lambda_1 x}$; $y_2' = x e^{\lambda_1 x}$ $\rightarrow$ studied later.

Assume $y_1 = y_2 \Rightarrow \Delta = 0 \Rightarrow a^2 - 4b = 0 \Rightarrow a^2 = 4b \Rightarrow b = (\frac{a}{2})^2$

$y_1 = e^{\lambda_1 x}$, $y_2 = xe^{+\lambda_1 x}$; $\lambda_1 = -\frac{a}{2}$

$y'' + ay' + by = 0$

$y_2 = xe^{\lambda_1 x} \rightarrow y_2' = e^{\lambda_1 x} + \lambda_1 x e^{\lambda_1 x}$

$y_2'' = \lambda_1 e^{\lambda_1 x} + \lambda_1 (e^{\lambda_1 x} + \lambda_1 x e^{\lambda_1 x})$

$= \lambda_1 e^{\lambda_1 x} + \lambda_1 e^{\lambda_1 x} + \lambda_1^2 x e^{\lambda_1 x} = 2\lambda_1 e^{\lambda_1 x} + \lambda_1^2 x e^{\lambda_1 x}$

$y'' + ay' + by = 0$:

$2\lambda_1 e^{\lambda_1 x} + \lambda_1^2 x e^{\lambda_1 x} + a e^{\lambda_1 x} + a\lambda_1 x e^{\lambda_1 x} + bxe^{\lambda_1 x} = 0$

$(2\lambda_1 + a) e^{\lambda_1 x} + (\lambda_1^2 + a\lambda_1 + b) x e^{\lambda_1 t} = 0$

Since $y_1 = -\frac{a}{2}$

$\underbrace{(-a+a)}_{0} e^{\lambda_1 x} + \underbrace{\left[\frac{a^2}{4} + a(-\frac{a}{2}) + \frac{a^2}{4}\right]}_{0} x e^{\lambda_1 x} = 0$

So $y_2 = xe^{\lambda_1 x}$ $(y_1 = e^{\lambda_1 x})$

Then check Wronskian for $y_1 = e^{\lambda_1 x}$; $y_2 = xe^{\lambda_2 x}$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^{\lambda_1 x} & xe^{+\lambda_1 x} \\ \lambda_1 e^{\lambda_1 x} & e^{\lambda_1 x} + \lambda_1 x e^{\lambda_1 x} \end{vmatrix}$$

$= e^{\lambda_1 x}(e^{\lambda_1 x} + \lambda_1 x e^{\lambda_1 x}) - \lambda_1 x e^{\lambda_1 x} e^{\lambda_1 x}$

$= e^{2\lambda_1 x} \neq 0$ So are y_1 & y_2 are independent.

Hence

$y = c_1 y_1 + c_2 y_2$

$= c_1 e^{\lambda_1 x} + c_2 x e^{\lambda_1 x}$

is also a solution for the homogeneous part

$$\underbrace{(\lambda^2 + a\lambda + b)}_{\text{Characteristic equation (Auxiliary equation)}} e^{\lambda t} = 0$$

$$\lambda_{1,2} = \frac{-a \mp \sqrt{a^2 - 4b}}{2}$$

Three possibilities :

i. $a^2 - 4b > 0$: Two distinct real roots

ii. $a^2 - 4b = 0$: A real double root

iii. $a^2 - 4b < 0$: Complex conjugate roots.

Example: (Distinct Real Root)

$$y'' + y' - 2y = 0 \qquad y(0) = 4, \; y'(0) = -5$$

Assume $y = e^{\lambda x}$

$$\lambda^2 e^{\lambda x} + \lambda e^{\lambda x} - 2e^{\lambda x} = 0$$

$$\Rightarrow (\lambda^2 + \lambda - 2) e^{\lambda x} = 0$$

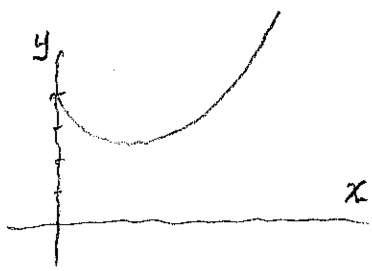
$$\lambda_{1,2} = \frac{-1 \mp \sqrt{1+8}}{2} = \frac{-1 \mp 3}{2} \; \begin{cases} 1 \\ -2 \end{cases}$$

$$y_1 = e^{x}, \quad y_2 = e^{-2x}$$

$$y = c_1 e^{x} + c_2 e^{-2x} \Rightarrow y' = c_1 e^{x} - 2c_2 e^{-2x}$$

$$y(0) = 4 \Rightarrow c_1 + c_2 = 4$$
$$y'(0) = -5 \Rightarrow c_1 - 2c_2 = -5$$
$$3c_1 = 3 \rightarrow c_1 = 1$$
$$c_2 = 3$$

Then $y = e^{x} + 3e^{-2x}$



Example: Double root. $\Delta = a^2 - 4b = 0$

$$\lambda_{1,2} = \frac{-a \mp \sqrt{a^2-4b}}{2} = -\frac{a}{2} \qquad b = \frac{a^2}{4}$$

Then $y_1 = e^{-\frac{a}{2}x}$

is a solution. $y_1' = -\frac{a}{2} e^{-\frac{a}{2}x}$; $y_1'' = \frac{a^2}{4} e^{-\frac{a}{2}x}$

$$y'' + ay' + by = 0$$

$$\frac{a^2}{4} e^{-\frac{a}{2}x} - \frac{a^2}{2} e^{-\frac{a}{2}x} + \frac{a^2}{4} e^{-\frac{a}{2}x} = 0$$

The second solution:

$$y_2 = x e^{-\frac{a}{2}x} \rightarrow y_2' = e^{-\frac{a}{2}x} - \frac{a}{2} x e^{-\frac{a}{2}x}$$

$$y'' = -\frac{a}{2} e^{-\frac{a}{2}x} - \frac{a}{2} e^{-\frac{a}{2}x} + \frac{a^2}{4} x e^{-\frac{a}{2}x}$$

$$y_2'' + a y_2' + b y_2 = 0$$

$$\underline{-ae^{-\frac{a}{2}x}} + \underline{\frac{a^2}{4} x e^{-\frac{a}{2}x}} + \underline{ae^{-\frac{a}{2}x}} - \underline{\frac{a^2}{2} x e^{-\frac{a}{2}x}} + \underline{\frac{a^2}{4} e^{-\frac{a}{2}x}} = 0$$

So $y_2 = x e^{-ax/2}$ is also a solution

Then total general solution:

$$y = c_1 y_1 + c_2 y_2$$

$$= c_1 e^{-\frac{a}{2}x} + c_2 x e^{-\frac{a}{2}x}$$

Example:

$$y'' + 6y' + 9 = 0 \quad ; \quad y = e^{\lambda x}$$

$$\lambda^2 + 6\lambda + 9 = 0 \rightarrow (\lambda+3)^2 = 0 \qquad \lambda_{1,2} = e^{-3x}$$

$\lambda_1 = -3 \Rightarrow y_1 = e^{-3x}$

$y_2 = x e^{-3x}$

General solution:

$$y = C_1 e^{-3x} + C_2 x e^{-3x}$$

Case III : Complex conjugate roots.

$$\Delta = a^2 - 4b < 0 \rightarrow 4b > a^2 \Rightarrow \sqrt{\Delta} = \sqrt{a^2-4b}$$

$$y_{1,2} = \frac{-a \mp \sqrt{a^2-4b}}{2} = -\frac{a}{2} \mp i\frac{\sqrt{4b-a^2}}{2} \quad ; \text{ Let } \omega = \frac{\sqrt{4b-a^2}}{2}$$
$$= -\frac{a}{2} \mp i\omega$$

$$y_1 = e^{\lambda_1 x} = e^{-\frac{a}{2}x} \cdot e^{i\omega x} = e^{-\frac{a}{2}x}(\cos\omega x + i\sin\omega x)$$

$$y_2 = e^{\lambda_2 x} = e^{-\frac{a}{2}x} \cdot e^{-i\omega x} = e^{-\frac{a}{2}x}(\cos\omega x - i\sin\omega t)$$

Then homogeneous solution ;

$$y = c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x}$$
$$= e^{-\frac{a}{2}x}\left[\underbrace{(c_1+c_2)}_{A}\cos\omega x + \underbrace{i(c_1-c_2)}_{B}\sin\omega x\right]$$
$$= e^{-\frac{a}{2}x}\left[A\cos\omega x + B\sin\omega x\right]$$

Example :

$$y'' + 0.4y' + 9.04 = 0 \quad ; \quad y(0) = 0, \; y'(0) = 3$$

Characteristic equation :

$$\lambda^2 + 0.4\lambda + 9.04 = 0$$
$$\lambda_{1,2} = \frac{-0.4 \mp \sqrt{0.16 - 36.16}}{2} = \frac{-0.4 \mp i6}{2} = -0.2 \mp i3$$

Homogeneous solution :

$$y = e^{-0.2x}(A\cos 3x + B\sin 3x) \Rightarrow$$

$$y(0) = 0 \Rightarrow 0 = 1(A + B\cdot 0) \Rightarrow A = 0 \rightarrow y = e^{-0.2x} B\sin 3x$$
$$= Be^{-0.2x}\sin 3x$$

$$y' = B[-0.2e^{-0.2x}\sin 3x + e^{-0.2x}(+3)\cos 3x]$$

$$y'(0) = 3 \Rightarrow 3 = B[0 + (1)(+3)(1)] \Rightarrow 3 = B3 \Rightarrow B = 1$$

Then $y = e^{-0.2x}\sin 3x$

Linear Second Order ODEs (With constant coefficients)

$$y'' + ay' + by = r(x)$$

Step 1 : Let $r(x) = 0$ and find homogeneous solution y_h -
$y_h = c_1\lambda_1 + c_2\lambda_2$; $y_h = (c_1 + c_2 x)e^{\lambda_1 x}$; $y_h = e^{-\frac{a}{2}x}(c_1\cos\omega t + c_2\sin\omega_2 t)$

Step 2 : Let $r(x)$ is a given. If $r(x)$ is an exponential, power of x, sine or cosine, or sums or products of these function, using the method so called "Method of undetermined coefficients", determine the nonhomogeneous solution y_p (particular solution for the $r(x)$ function),

Step 3 : Obtain the general solution as
$$y = y_h + y_p$$

Step 4 : Determine c_1 & c_2 using the initial conditions.

Homogeneous solution

Case	$\Delta = a^2 - 4dc$	Roots	Roots	Homogeneous solution
I	$\Delta > 0$	Distinct real	$\lambda_1 = (-a + \sqrt{\Delta})/2$ $\lambda_2 = (-a - \sqrt{\Delta})/2$	$c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x}$
II	$\Delta = 0$	Real double root	$\lambda_1 = -a/2$	$(c_1 + c_2 x)e^{\lambda_1 x}$
II	$\Delta < 0$	Complex conjugate roots	$\omega = \sqrt{\Delta}/2$	$e^{-ax/2}(c_1\cos\omega t + c_2\sin\omega t)$

Method of Undetermined Coefficients

$r(x)$	Choice for y_p	Use Linear ODE
$ke^{\gamma x}$	$Ce^{\gamma x}$	Solve for C
kx^n	$K_n x^n + K_{n-1}x^{n-1} + \dots + K_0$	Solve for $K_n, K_{n-1}, \dots, K_0$
$k\cos\omega_0 x$ $k\sin\omega_0 x$	$K\cos\omega_0 x + M\sin\omega_0 x$	M & M
$ke^{\alpha x}\cos\omega_0 x$ $ke^{\alpha x}\sin\omega_0 x$	$e^{\alpha x}(K\cos\omega_0 x + M\sin\omega_0 x)$	K & M

Example: $y'' + y = 0.001x^2 \quad y(0) = 0,\ y'(0) = 1.5$

Step 1 : Homogenenoous solution

$$y'' + y = 0$$

$$\lambda^2 + 1 = 0 \Rightarrow \lambda_{1,2} = \mp i \rightarrow \omega = 1,\ a = 0$$

$$y_h = C_1 \cos x + C_2 \sin x$$

Step 2 : Nonhomogeneous solution y_p for $r(x) = 0.001x^2$

Assume

$$y_p = K_2 x^2 + K_1 x + K_0$$

$$y_p' = 2K_2 x + K_1$$

$$y_p'' = 2K_2$$

Substituting these into

$$y'' + y = 0.001x^2$$

$$2K_2 + K_2 x^2 + K_1 x + K_0 = 0.001x^2$$

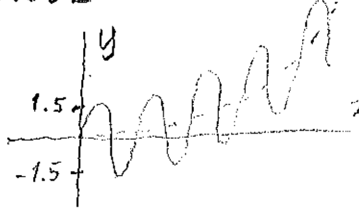
$$\underbrace{K_2}_{=0.001} x^2 + \underbrace{K_1}_{=0} x + \underbrace{(2K_2 + K_0)}_{=0} = 0.001x^2$$

$$\Rightarrow K_2 = 0.001$$

$$K_1 = 0$$

$$2K_2 + K_0 = 0 \Rightarrow K_0 = -2K_2 = -0.002$$

Then $\quad y_p = 0.001x^2 - 0.002$



Step 3 : $y = y_h + y_p =$

$$= C_1 \cos x + C_2 \sin x + 0.001x^2 - 0.002$$

Step 4 : Determine C_1 & C_2 using initial conditions

$$y(0) = 0 \Rightarrow 0 = C_1 + 0 + 0 - 0.002 \Rightarrow \boxed{C_1 = 0.002}$$

$$y = 0.002\cos x + C_2 \sin x + 0.001x^2 - 0.002$$

$$y' = -0.002\sin x + C_2 \cos x + 0.002x$$

$$y'(0) = 1.5 \Rightarrow 1.5 = 0 + C_2 \Rightarrow \boxed{C_2 = 1.5}$$

$$y = 0.002\cos x + 1.5\sin x + 0.001x^2 - 0.002$$

Example: $y''+3y'+2.25y=-10e^{-x}$ $y(0)=1,\ y'(0)=0$

Step 1 : Homogeneous solution

$$y''+3y'+2.25y=0$$

$$\lambda^2+3\lambda+2.25=0$$

$$\Delta = 9-(4)(2.25)=0 \Rightarrow \lambda_1=-\frac{3}{2}=-1.5$$

Then

$$y_h=(C_1+C_2x)e^{-1.5x}$$

Step 2 : Nonhomogeneous solution for $r(x)=-10e^{-x}$

$$y_p=Ke^{-x}$$

$$y_p'=-Ke^{-x}$$

$$y_p''=Ke^{-x}$$

$$y''+3y'+2.25y=-10e^{-x}$$

$$Ke^{-x}-3Ke^{-x}+2.25Ke^{-x}=-10e^{-x}$$

$$0.25Ke^{-x}=-10e^{-x} \Rightarrow 0.25K=-10 \Rightarrow K=-40$$

$$y_p=-40e^{-x}$$

Step 3 : Total solution :

$$y=y_h+y_p=(C_1+C_2x)e^{-1.5x}-40e^{-x}$$

Step 4 : C_1 & C_2

$y(0)=1 \Rightarrow 1=C_1-40 \Rightarrow C_1=41$

$$y=(41+C_2x)e^{-1.5x}-40e^{-x}$$

$$y'=C_2e^{-1.5x}-1.5(41+C_2x)e^{-1.5x}+40e^{-x}$$

$y'(0)=0 \Rightarrow 0=C_2 \quad -1.5(41) \quad +40=$

$$0=C_2-21.5 \Rightarrow C_2=21.5$$

Then $y=(41+21.5x)e^{-1.5x}-40e^{-x}$

Example : $y'' + 3y' + 2.25y = -10e^{-1.5x}$ $\quad y(0) = 1,\ y'(0) = 0$

Step 1 : Homogeneous solution :

$$y_h = (c_1 + c_2 x)e^{-1.5x}$$

Step 2 : Nonhomogeneous solution for $r(x) = -10e^{-1.5x}$

$$y_p = Ke^{-1.5x} \quad \times \text{ exists in } y_h$$

$$= Kxe^{-1.5x} \quad \times \text{ exists in } y_h$$

$$= Kx^2 e^{-1.5x} \quad \checkmark$$

$$y_p' = K\,2x\,e^{-1.5x} - 1.5Kx^2 e^{-1.5x} = K(2x - 1.5x^2)e^{-1.5x}$$

$$y_p'' = K\left[(2 - 3x)e^{-1.5x} - 1.5(2x - 1.5x^2)e^{-1.5x}\right]$$

$$= K(2 - 3x - 3x + 2.25x^2)e^{-1.5x}$$

$$y_p'' = K(2 - 6x + 2.25x^2)e^{-1.5x}$$

$$y'' + 3y' + 2.25y = -10e^{-1.5x}$$

$$K(2 - 6x + 2.25x^2)\cancel{e^{-1.5x}} + 3K(2x - 1.5x^2)\cancel{e^{-1.5x}} + 2.25Kx^2\cancel{e^{-1.5x}} = -10\cancel{e^{-1.5x}}$$

$$\underbrace{[K(2.25) - 4.5K + 2.25K]}_{0}x^2 + \underbrace{[\cancel{-6K} + \cancel{6K}]}_{0}x + 2K = -10$$

$$2K = -10 \Rightarrow \boxed{K = -5}$$

Step 3 : $y_t = (c_1 + c_2 x)e^{-1.5x} + Kx^2 e^{-1.5x}$

$$= (c_1 + c_2 x - 5x^2)e^{-1.5x}$$

Step 4 : Initial conditions for determining c_1 & c_2 :

$$y(0) = 1 \Rightarrow 1 = c_1 \Rightarrow \boxed{c_1 = 1}$$

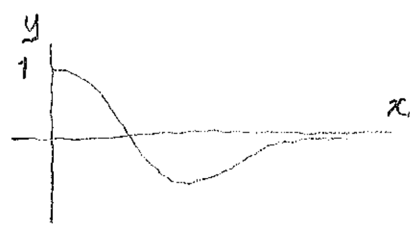
$$y = (1 + c_2 x - 5x^2)e^{-1.5x}$$

$$y' = (c_2 - 10x)e^{-1.5x} - 1.5(1 + c_2 x - 5x^2)e^{-1.5x}$$

$$y'(0) = 0 \Rightarrow 0 = c_2 - 1.5 \Rightarrow \boxed{c_2 = 1.5}$$

Hence

$$y = (1 + 1.5x - 1.5x^2)e^{-1.5x}$$



Example

$$y'' + 2y' + 0.75y = 2\cos x - 0.25\sin x + 0.09x$$

$$y(0) = 2.78, \quad y'(0) = -0.43$$

Step 1: Homogeneous solution:

$$y'' + 2y'' + 0.75y = 0$$

$$\lambda^2 + 2\lambda + 0.75 = 0$$

$$\Delta = 4 - (4)(0.75) = 1 \Rightarrow \lambda_{1,2} = \frac{-2 \mp 1}{2} \begin{cases} -3/2 \\ -1/2 \end{cases}$$

Then

$$y_h = c_1 e^{-\frac{x}{2}} + c_2 e^{-\frac{3}{2}x}$$

step 2: Nonhomogeneous solution for

$$r(x) = 2\cos x - 0.25\sin x + 0.09x$$

Nonhomogeneneous solution will have two components

$$\underbrace{y_{p_1} = K\cos x + M\sin x}_{\text{For sine\&cosine}}, \quad y_{p_2} = K_1 x + K_0 \quad \text{For } 0.09x$$

$$y_p = K\cos x + M\sin x + K_1 x + K_0$$

$$y_p' = -K\sin x + M\cos x + K_1$$

$$y_p'' = -K\cos x - M\cos x$$

$$y'' + 2y' + 0.75y = 2\cos x - 0.25\sin x + 0.09$$

$$(-K\cos x - M\sin x) + 2(-K\sin x + M\cos x + K_1) + 0.75(K\cos x + M\sin x + K_1 x + K_0)$$

$$= 2\cos x - 0.25\sin x + 0.09x$$

$$-K + 2M + 0.75K = 2$$

$$-M - 2K + 0.75M = -0.25$$

$$-0.25K + 2M = 2$$

$$-2K \quad -0.25M = -0.25$$

$$0.75K_1 = 0.09 \Rightarrow \boxed{K_1 = 0.12}$$

$$2K_1 + 0.75K_0 = 0 \Rightarrow K_0 = \frac{-0.24}{0.75}$$

$$\boxed{K_0 = -0.32}$$

$$\left[\begin{array}{cc|c} -0.25 & 2 & 2 \\ -2 & -0.25 & -0.25 \end{array}\right] \xrightarrow{8R_1 - R_2 \to R_2} \left[\begin{array}{cc|c} -0.25 & 2 & 2 \\ 0 & 16.25 & 16.25 \end{array}\right] \quad \boxed{M = 1}$$

$$-0.25K + 2 = 2 \Rightarrow \boxed{K = 0}$$

$$y_p = K\cos x + M\sin x + K_1 x + K_0$$

$$= \sin x + 0.12x - 0.32$$

Totol solution

$$y = y_h + y_p$$

$$y = c_1 e^{-\frac{x}{2}} + c_2 e^{-\frac{3}{2}x} + \sin x + 0.12x - 0.32$$

$$y(0) = 2.78 \Rightarrow 2.78 = c_1 + c_2 + 0 + 0 - 0.32 \Rightarrow c_1 + c_2 = 3.1$$

$$y' = -\frac{c_1}{2} e^{-\frac{x}{2}} - \frac{3}{2} c_2 + \cos x + 0.12$$

$$y'(0) = -0.43 \Rightarrow -0.43 = -\frac{1}{2} c_1 - \frac{3}{2} c_2 + 1 + 0.12 \Rightarrow$$

$$-\frac{1}{2} c_1 - \frac{3}{2} c_2 = -1.55 \Rightarrow -c_1 - 3c_2 = -3.1$$

$$-2c_2 = 0 \Rightarrow \boxed{c_2 = 0} \qquad c_1 + c_2 = 3.1 \Rightarrow \boxed{c_1 = 3.1}$$

Therefore

$$y = 3.1 e^{-\frac{x}{2}} + \sin x + 0.12x - 0.32$$