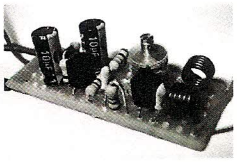
İZMİR UNIVERSITY OF ECONOMICS
Department of Electrical and Electronics Engineering

EEE 331 Analog Electronics
Fall 2015/2016



FINAL EXAM

Jan 5, 2016
150 min

SOLUTIONS

INSTRUCTIONS

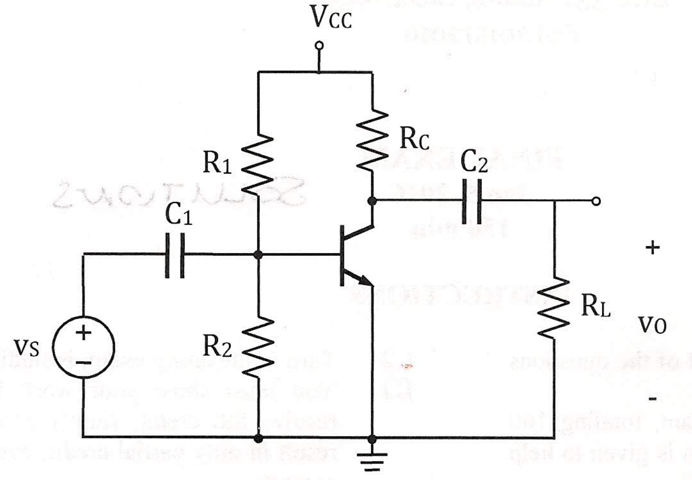
- Read all of the instructions and all of the questions before beginning the exam.
- There are 5 questions on this exam, totaling 100 points. The credit for each problem is given to help you allocate your time accordingly.
- Do not spend all your time on one problem and on one part and attempt to solve all of them.
- Calculators are allowed, but borrowing is not allowed.
- Your mobile phones must be turned off during the exam.
- Turn in the entire exam, including this cover sheet.
- You must show your work for all problems to receive full credit; simply providing answers will result in only partial credit, even if the answers are correct.
- Please indicate the number of page where your work is to be continued.
- Put your name on any additional material that you submit.
- Be sure to provide units where necessary.

		Question	Points	Grade
Last Name	:........................	1	20	
Name	:........................	2	20	
Section	:........................	3	20	
Student No	:........................	4	20	
		5	20	
		TOTAL	100	

The basic equations of the output characteristics of an NMOS transistor

V_{GS}	V_{DS}	I_D
i) $V_{GS} < V_{Tn}$	-	0
ii) $V_{GS} > V_{Tn}$	a) $V_{DS} < V_{GS} - V_{Tn}$	$K_n\left[2(V_{GS}-V_{Tn})V_{DS}-V_{DS}^2\right]$
	b) $V_{GS} - V_{Tn} \leq V_{DS}$	$K_n(V_{GS}-V_{Tn})^2$
where $K_n = \frac{K'_n}{2}\left(\frac{W}{L}\right)$ and $K'_n = \mu_n C_{ox}$		

Q1. (20 pts) Consider the following BJT amplifier.



Circuit Parameters

$V_{CC} = 15$ V

$R_1 = 400$ kΩ

$R_2 = 220$ kΩ

$R_C = 2$ kΩ

$R_L = 40$ kΩ

$C_1 = 50$ μF

$C_2 = 50$ μF

Transistor Parameters

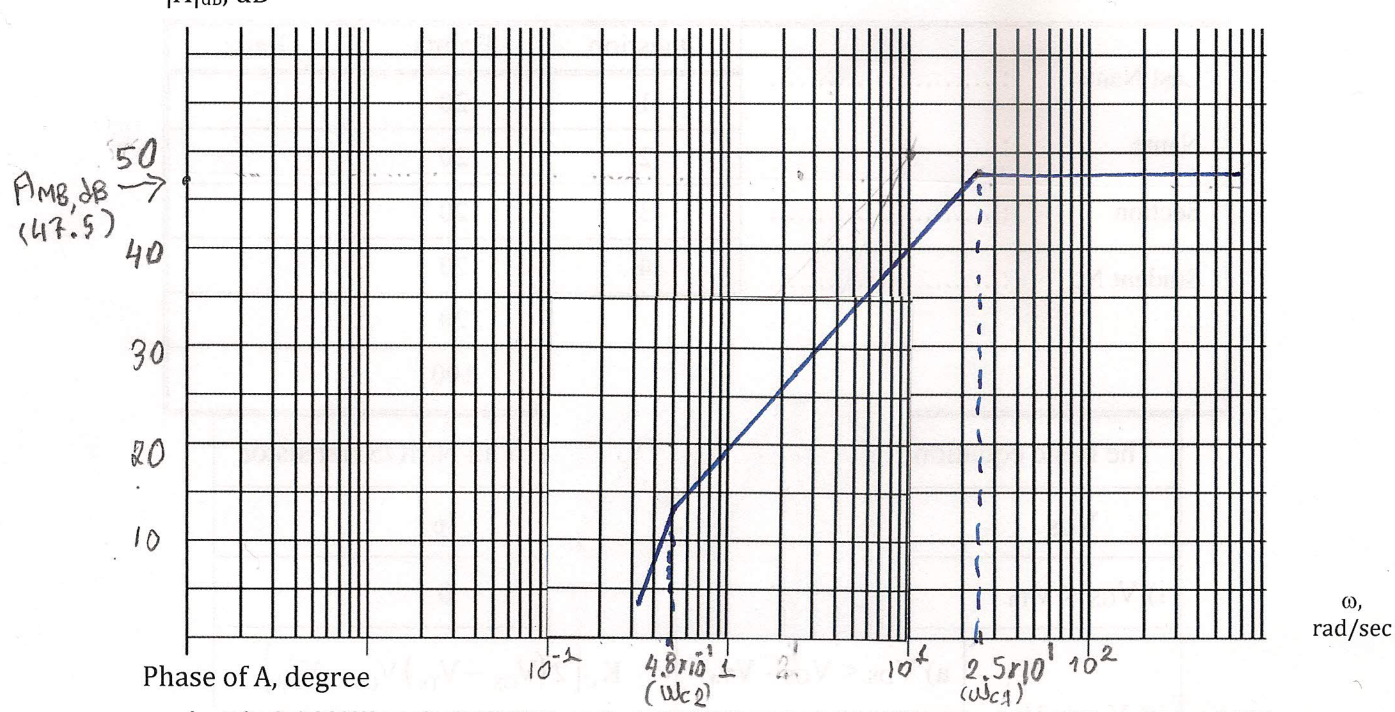
$V_{BE(ON)} = 0.7$ V

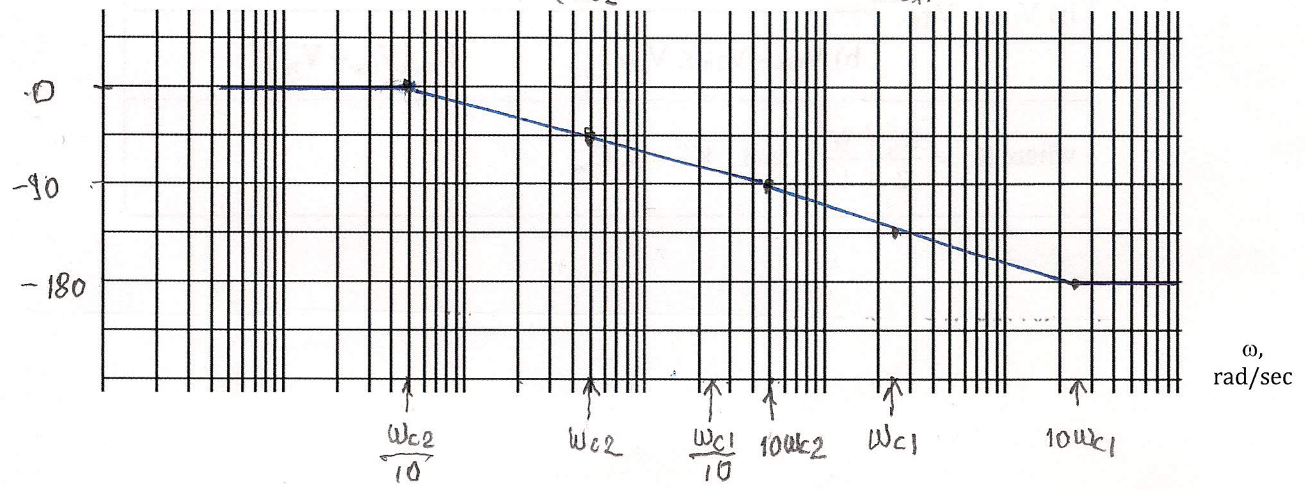
$V_T = 26$ mV

$\beta = 100$

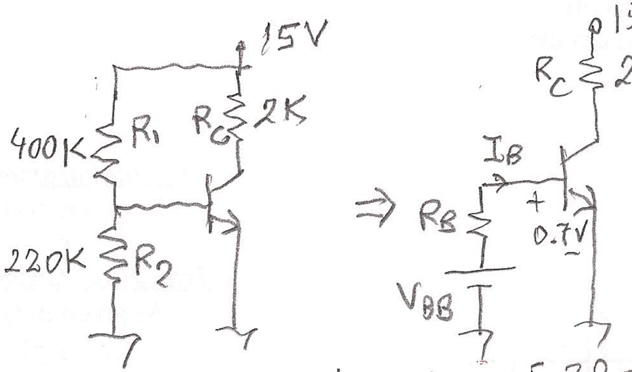
$V_A = \infty$

i. Determine the small signal parameters and draw the low frequency small signal model.
ii. Determine the lower corner frequency.
iii. Find the midband voltage gain.
iv. Plot the low frequency magnitude response.





(a) To determine small signal parameters, DC analysis is needed.



$V_{BB} = \frac{R_2}{R_1+R_2} V_{CC} = \frac{220}{620} \cdot 15\,V$

$= 5.32\,V$

$R_B = R_1 \parallel R_2 = 141.9K$

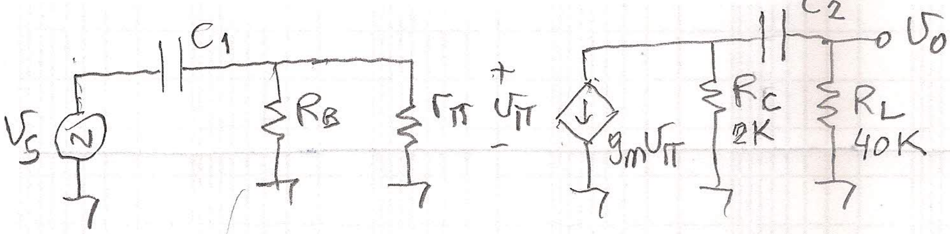
$I_{BQ} = \frac{V_{BB} - V_{BE(ON)}}{R_B} = \frac{5.32-0.7}{141.9} = 32.56\,\mu A \Rightarrow I_{CQ} = 3.256\,mA$

$V_{CEQ} = V_{CC} - R_C I_{CQ} = 8.49\,V$ ($V_{CEQ} > V_{CE(SAT)}$: Active)

$r_\pi = \frac{V_T}{I_{BQ}} = \frac{26\,mA}{32.56\,\mu A} \cong 0.8\,K \Rightarrow g_m = \frac{\beta}{r_\pi} = \frac{100}{0.8K} = 125\,mS$

$r_o = \infty$

Low frequency small signal model:



$R_L' = R_C \parallel R_L$
$= 1.9K$

Two poles:

$\omega_{C1} = \frac{1}{R_{eq1} C_1}$; $R_{eq1} = R_B \parallel r_\pi = 141.9K \parallel 0.8K \cong 0.8K$

$\omega_{C1} = \frac{1}{(0.8K)(50\mu F)} = 25$ rad/sec $= 2.5 \times 10^1$ rad/sec

$\omega_{C2} = \frac{1}{R_{eq2} C_2}$; $R_{eq2} = R_C + R_L = 42K$

$\omega_{C2} = \frac{1}{(42K)(50\mu F)} = 0.48$ rad/sec $= 4.8 \times 10^{-1}$ rad/sec corner

Midband gain =

$V_\pi = V_s$; $V_o = -g_m (R_C \parallel R_L) V_\pi = -(125\,mS)(1.9K) = -237.5\,V_s$

$A_V = -237.5 \Rightarrow A_{MB,dB} = 47.5\,dB$

$A(j\omega) = \left(\frac{j\omega}{j\omega + \omega_{C1}}\right)\left(\frac{j\omega}{j\omega + \omega_{C2}}\right) A_{MB} = \frac{\omega^2 (237.5)}{(j\omega + \omega_{C1})(j\omega + \omega_{C2})}$

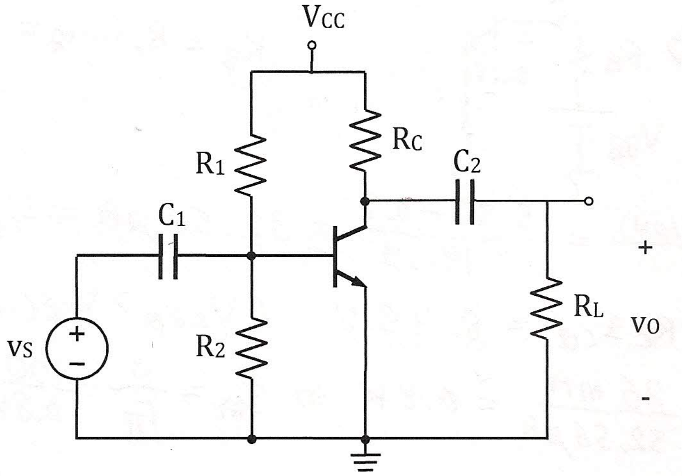
i. Magnitude plot with two poles.

ii. $\theta(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_{C1}}\right) - \tan^{-1}\left(\frac{\omega}{\omega_{C2}}\right)$

plots are on the upper page.

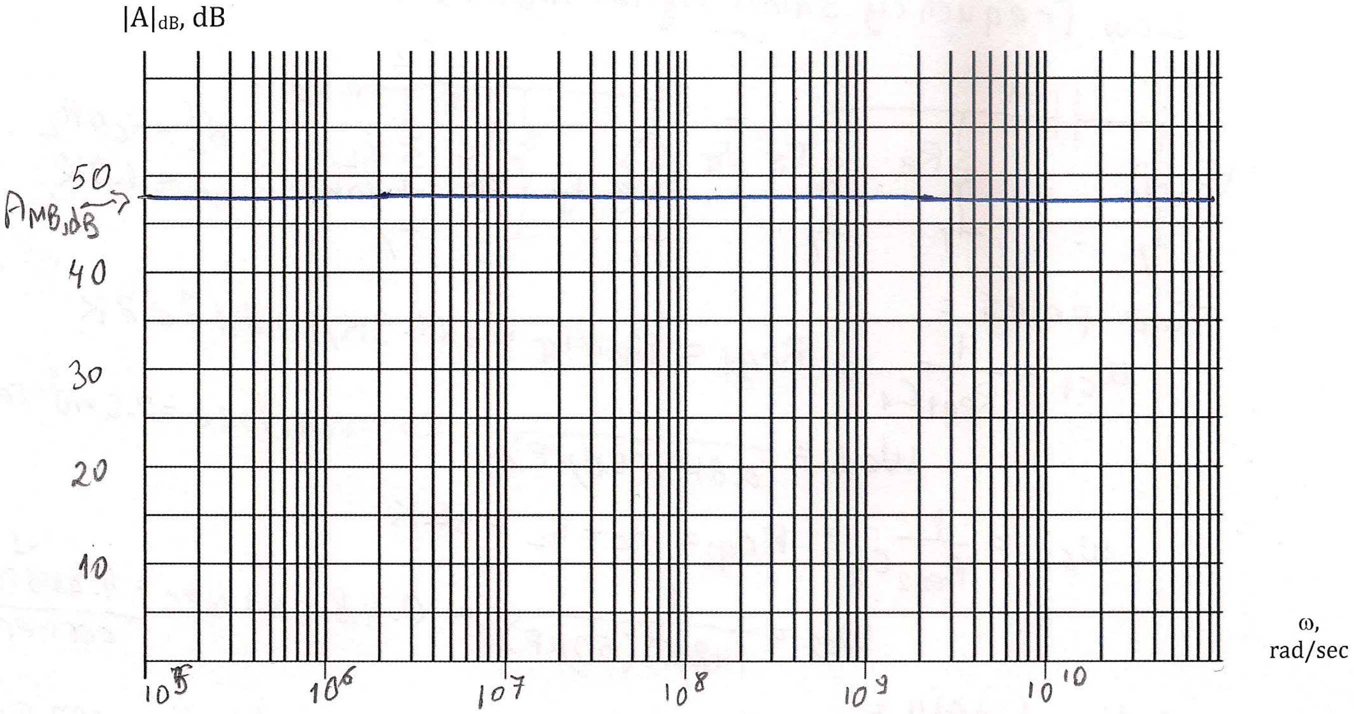
Q2. (20 pts) Consider the BJT amplifier circuit given in Question 1 above.

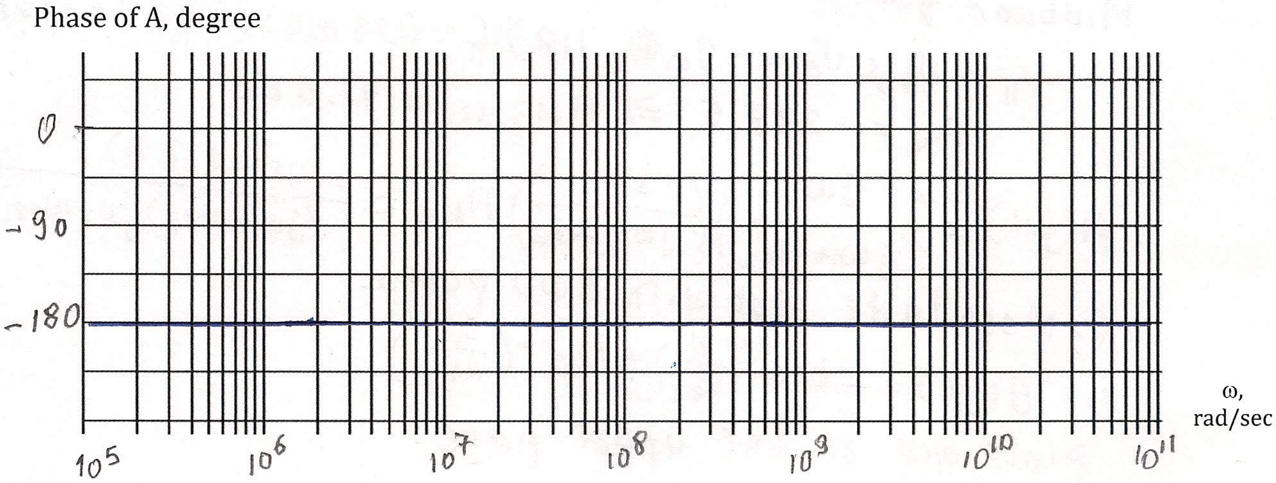
i. Draw the high-frequency equivalent circuit.
ii. Simplify the high-frequency equivalent circuit.
iii. Determine the higher corner frequency.
iv. Plot the overall frequency response.



Circuit Parameters
As given in Q1.

Transistor Parameters
As given in Q1.
$C_\pi = 1$ pF
$C_\mu = 1$ pF



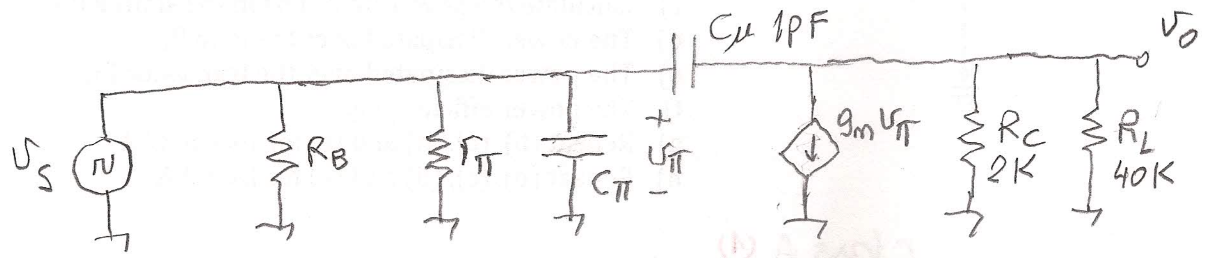


(i) From Q_1:

$g_m = 125$ mS

$A_{MB} = -237.5$

High frequency small signal model
(External capacitors C_1 & C_2 are short)



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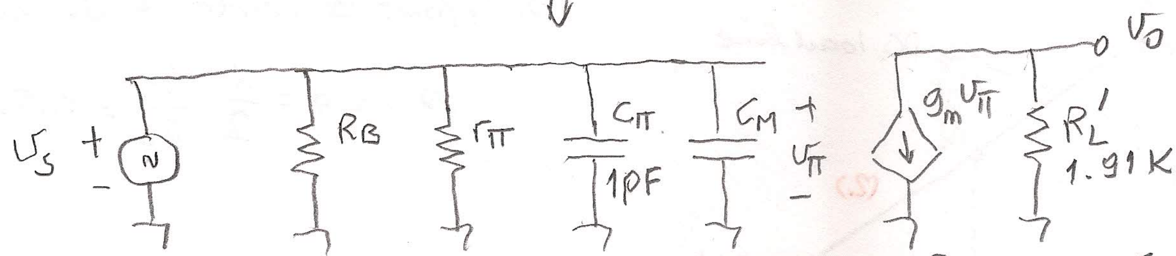


$$C_M = C_\mu(1+g_m R_L') = 1pF[1+(125)(1.91)] = 239.6\ pF$$

$$C_T = C_\pi + C_M = 240.6\ pF$$

In the high frequency region =(with single pole)

$$A(j\omega) = \frac{1}{1+j\frac{\omega}{\omega_H}} \cdot A_{MB} =$$

$A_{MB} = -237.5$

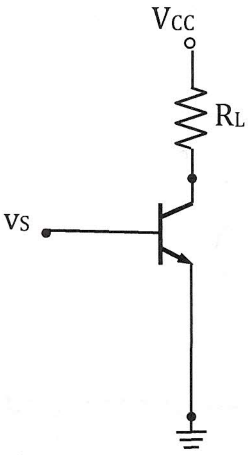
$\omega_H = \frac{1}{R_{eq} C_T}$; $R_{eq} = 0$ (since v_s is short)

then $\omega_H = \infty$ and $A(j\omega)$ reduces to

$A(j\omega) = -237.5$

$\theta(j\omega) = -180^\circ$

Q3. (20 pts)



For the power amplifier given with

V_{CC} = 15 V, R_L = 15 Ω and $V_{CE(SAT)}$ = 0 V.

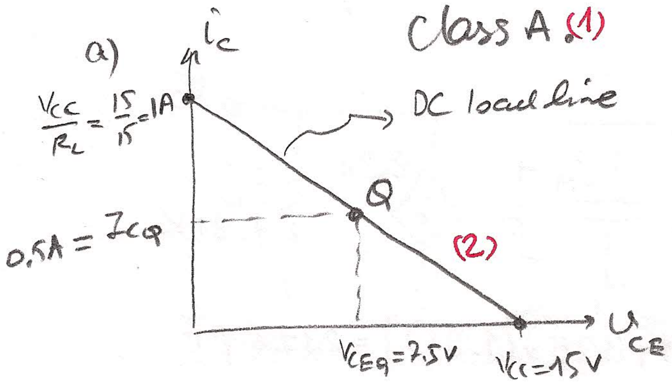
a) Draw the DC load line and determine the class of the power amplifier.
b) Determine I_{CQ} assuming the Q-point is at the center of the DC load line and

Now consider

$i_c(t) = I_{CQ} + I_{CP} \sin \omega t$ and I_{CP} = 0.5 A.

c) Calculate the power driven from the source P_{CC},
d) The power dissipated over the load R_L,
e) The power dissipated over the transistor P_T,
f) The power efficiency η.
g) Repeat (b), (c), (d) and (e) for I_{CP} = 0.25 A.
h) Repeat (b), (c), (d) and (e) for I_{CP} = 0 A.

a)



b) Q point @ center of DC load line

$\Rightarrow I_{CQ} = \frac{1}{2} \frac{V_{CC}}{R_L} = 0.5A$. (1)

c) $P_{CC} = V_{CC} \cdot I_{CQ} = (15)(0.5) = 7.5\,W$ (2)

d) $P_{L,DC} = R_L \cdot I_{CQ}^2 = (15)(0.5)^2 = 3.75\,W$ (2)

$P_{L,AC} = \frac{1}{2} R_L I_{CP}^2 = \frac{1}{2}(15)(0.5)^2 = 1.875\,W$ (2)

e) $P_T = P_{CC} - P_L = 7.5 - 3.75 - 1.875 = 1.875\,W$ (2)

f) $\eta = \frac{P_{L,AC}}{P_{CC}} = \frac{1.875}{7.5} = 25\%$ (2) → (highest power efficiency achievable in class A power amps)

g) $P_{L,AC} = \frac{1}{2}(15)(0.25)^2 = 0.469\,W$

$P_T = 7.5 - 3.75 - 0.469 \cong 3.28\,W$ (3)

$\eta = \frac{0.469}{7.5} = 0.0625 = 6.25\%$

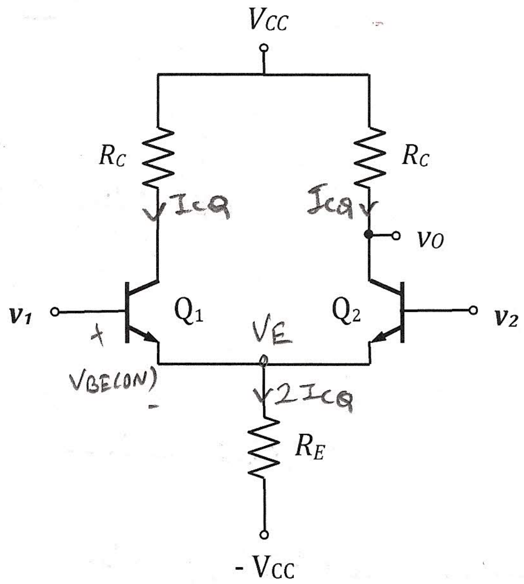
h) $P_{L,AC} = 0\ W$

(3) $P_T = 7.5 - 3.75 = 3.75 W \rightarrow$ highest transistor power (used to select the transistor)

$\eta = 0\%$ $\rightarrow$ lowest power efficiency.

Q4. (20 pts) Consider the differential amplifier given below. The output v_O is taken from the collector of Q_2.

(a) Determine the value of R_E to set I_{CQ} as given below. (Check whether the transistors are in forward active or not.)
(b) Determine the DC value of the output voltage V_{OQ}.
(c) Find the differential-mode gain $A_{dm} = v_O/v_{dm}$.
(d) Find the common-mode gain $A_{cm} = v_O/v_{cm}$.
(e) Find CMRR.
(f) If $v_1 = 2\sin(2\pi*75t) + 0.2\sin(2\pi*1000t)$ volts, and
$v_2 = 2\sin(2\pi*75t) - 0.2\sin(2\pi*1000t)$ volts,
find the total voltage v_O.



Circuit Parameters

$V_{CC} = 12$ V
$R_C = 2$ K
$I_{CQ} = 5$ mA

Transistor Parameters

$\beta = 200$
$V_{BE(ON)} = 0.7$ V
$V_T = 26$ mV
$V_A = \infty$

a) $+V_{BE(ON)} + 2I_{CQ} \cdot R_E - V_{CC} = 0 \Rightarrow R_E = \dfrac{V_{CC} - V_{BE(ON)}}{2I_{CQ}} = \dfrac{12 - 0.7}{(2)(5m)} = 1.13\,k\Omega$

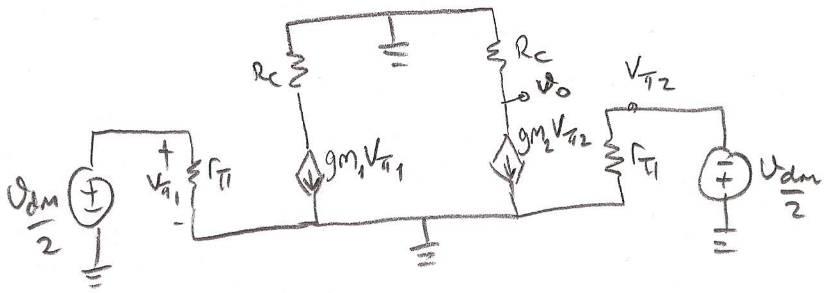
(3)

$V_C = V_{CC} - R_C I_{CQ} = 12 - (2k)(5m) = 2V$
$V_E = -V_{BE(on)} = -0.7V$
$\Rightarrow V_{CE} = 2 - (-0.7) = 2.7V > V_{CE(SAT)}$

Forward active ✓

b) $V_{OQ} = V_{CC} - R_C I_{CQ} = 2V$
(2)

c) Differential Mode Model:



(2)

$I_{BQ} = \dfrac{I_{CQ}}{\beta} = \dfrac{5mA}{200} = 0.025mA$

$r_\pi = \dfrac{V_T}{I_{BQ}} = \dfrac{26mV}{0.025mA} = 10.4\,k\Omega$

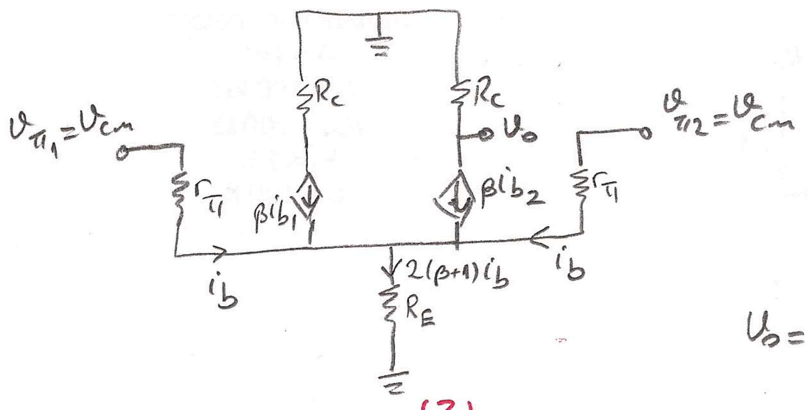
$g_m = \dfrac{\beta}{r_\pi} = \dfrac{200}{10.4K} = 19.2\,mA/V$

$v_o = -R_C g_m V_{\pi 2} = -(2K)(19.2mA/V)\left(-\dfrac{v_{dm}}{2}\right)$

$= 19.2\, v_{dm}$ (3)

$\boxed{A_{dm} = 19.2}$

d) Common-Mode Model:



(2)

$i_{b_1} = i_{b_2} = i_b.$

$$v_{cm} = r_\pi \cdot i_b + 2(\beta+1) i_b \cdot R_E$$
$$= [r_\pi + 2(\beta+1)R_E] \cdot i_b$$

$$v_o = -\beta R_C \cdot i_b = -\frac{\beta R_C}{r_\pi + 2(\beta+1)R_E} \cdot v_{cm}$$

$$= -\frac{(200)(2K)}{(10.4K) + 2(201)(1.13K)} v_{cm}$$

$$= \frac{-400}{464.66} v_{cm} = -0.86 v_{cm}.$$

(3)

$$\boxed{A_{cm} = -0.86}$$

e) $CMRR = \left|\frac{A_{dm}}{A_{cm}}\right| = \frac{19.2}{0.86} = 22.3 \Rightarrow CMRR_{dB} = 20 \log(22.3) = 26.97 \text{ dB}.$

(2)

f) $v_{dm} = v_1 - v_2 = 0.4 \sin(2\pi \times 1000t)$ Volts.

(3)

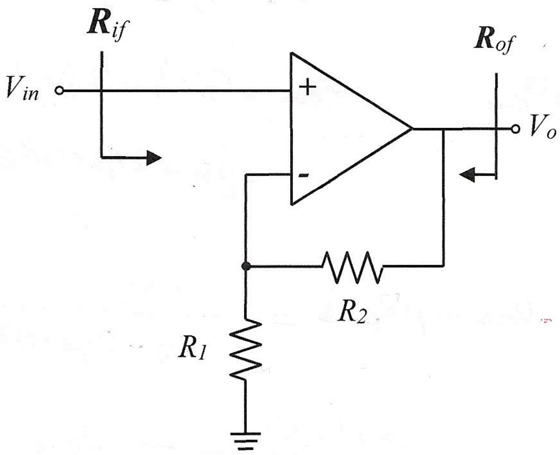
$v_{cm} = \frac{v_1 + v_2}{2} = 2 \sin(2\pi \times 75t)$ Volts.

$$v_o = V_{OQ} + A_{dm} v_{dm} + A_{cm} v_{cm}$$

$$= 2 + (19.2)(0.4 \sin 2\pi \times 1000t) - (0.86)(2 \sin 2\pi \times 75t)$$

$$= 2 + 7.68 \sin 2\pi \times 1000t - 1.72 \sin 2\pi \times 75t \text{ Volts.}$$

Q5. (20 pts) Consider the feedback amplifier given below.

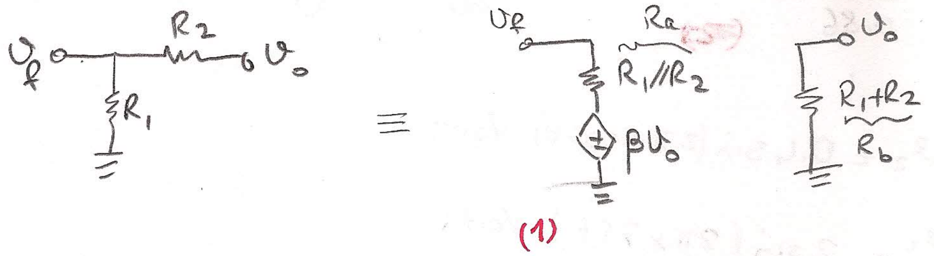


OPAMP Parameters
A = 10^3
R_{in} = 100 kΩ
R_{out} = 200 Ω
R_1 = 1 K
R_2 = 100 K

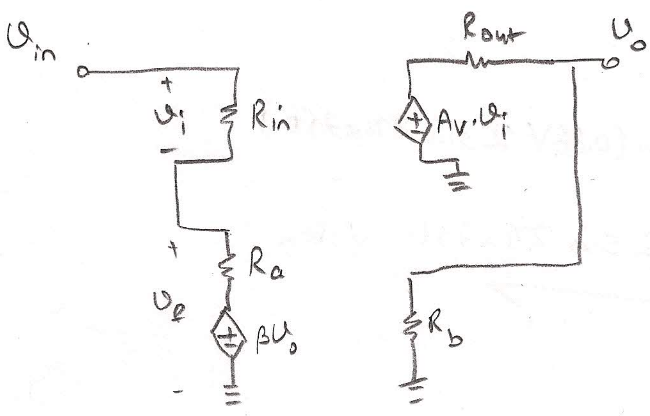
a) Identify the type of feedback,
b) Derive the expressions for the closed loop gain $A_{vf} = v_o/v_{in}$, *input resistance* R_{if} and output resistance R_{of}.
c) Compute the values of the closed loop gain A_{vf}, *input resistance* R_{if} and output resistance R_{of}.
d) Comment on the effect of feedback on the input and output resistances.
e) Now, assume OPAMP is ideal ($R_{in} = \infty$ and $R_{out} = 0$) and re-compute the closed loop gain A_{vf}.

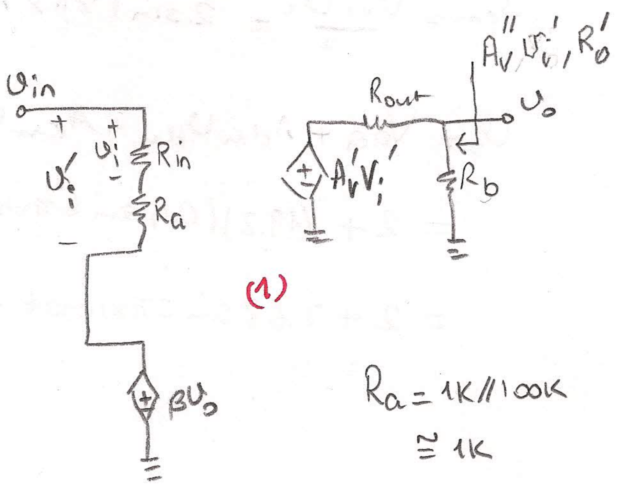
a) Series - shunt (3)

b) Feedback circuit model



$\beta = \frac{R_1}{R_1+R_2} = \frac{1K}{101K} \cong 0.01$ (1)





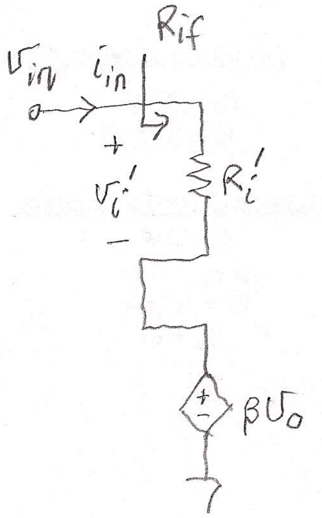
$R_a = 1K // 100K \cong 1K$

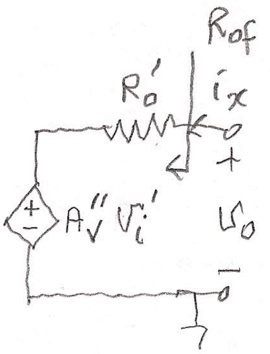
$R_b = 1K + 100K \cong 101K\Omega$

$v_i = \frac{R_{in}}{R_{in}+R_a} v_i'$

$A_v v_i = \underbrace{A_v \frac{R_{in}}{R_{in}+R_a}}_{A_v'} v_i' = A_v' v_i'$

$A_v' = 10^3 \frac{100}{100+1} = 0.99 \times 10^3$





$$A_v'' v_i' = \underbrace{\frac{R_b}{R_b+R_{out}}}_{A_v''} \cdot A_v' v_i'$$

$$A_v'' = \frac{101K}{101K+0.2K}\, 0.99\times10^3 \cong 0.99\times10^3$$

$$R_o' = R_b // R_{out}$$
$$= 101K//0.2K = 0.2K$$

$$R_i' = R_{in}+R_a = 100K + 1K = 101K$$

$$v_o = A_v'' v_i' = A_v''(v_{in}-\beta v_o) = A_v'' v_{in} - A_v''\beta v_o$$

$$(1+A_v''\beta)v_o = A_v'' v_{in} \Rightarrow A_{vf} = \frac{v_o}{v_{in}} = \frac{A_v''}{1+\beta A_v''}$$

$$R_{if} = \frac{v_{in}}{i_{in}} = \frac{v_i'+\beta v_o}{i_{in}} = \frac{v_i'+\beta A_v'' v_i'}{i_{in}} = (1+\beta A_v'')\frac{v_i'}{i_{in}} = (1+\beta A_v'')R_i'$$

$$R_{of} = \frac{v_o}{i_x}\Big|_{v_{in}=0} \;;\; v_o = R_o' i_x + A_v'' v_i'$$

$$v_i' = v_{in} - \beta v_o\Big|_{v_{in}=0} = -\beta v_o$$

$$\left.\begin{aligned} v_o &= R_o' i_x - A_v''\beta v_o \\ (1+A_v''\beta)v_o &= R_o' i_x \\ \frac{v_o}{i_x} &= \frac{R_o'}{1+A_v''\beta}\end{aligned}\right.$$

Hence $R_{of} = \dfrac{R_o'}{1+A_v''\beta}$

(c) $$A_{vf} = \frac{A_v''}{1+\beta A_v''} = \frac{0.99\times10^3}{1+(0.99\times10^3)(10^{-2})} = \frac{0.99\times10^3}{1+9.9} = 90$$

$$R_{if} = (1+\beta A_v'')R_i' = (1+9.9)(101K) = 1.1\,M\Omega$$

$$R_{of} = \frac{R_o'}{1+A_v''\beta} = \frac{0.2K}{1+9.9} = 18.3\,\Omega$$

(d) Input resistance is increased from 100K to 1.1 MΩ, whereas output resistance is decreased from 200Ω to 18.3Ω.

(e) $R_{in} = \infty$, $R_{out} = 0$

$$A_v'' = \frac{R_b}{R_b+R_{out}} A_v' = A_v' = \frac{R_{in}}{R_{in}+R_a} A_v = A_v = 10^3$$

$$R_{if} = (1+\beta A_v'')R_i' = (1+\beta A_v)(R_{in}+R_a) = \infty$$

$$R_{of} = \frac{R_o'}{1+\beta A_v''} = \frac{R_b//R_{out}}{1+\beta A_v''} = 0$$